Exploiting Dense Sensitivity Matrices in Linear Optimal Power Flow

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Outline

- Background
- Linear OPF formulations
- Why are dense constraints better for large-scale DCOPF?
- Modeling simplifications
- Preliminary results

Advanced Grid Research
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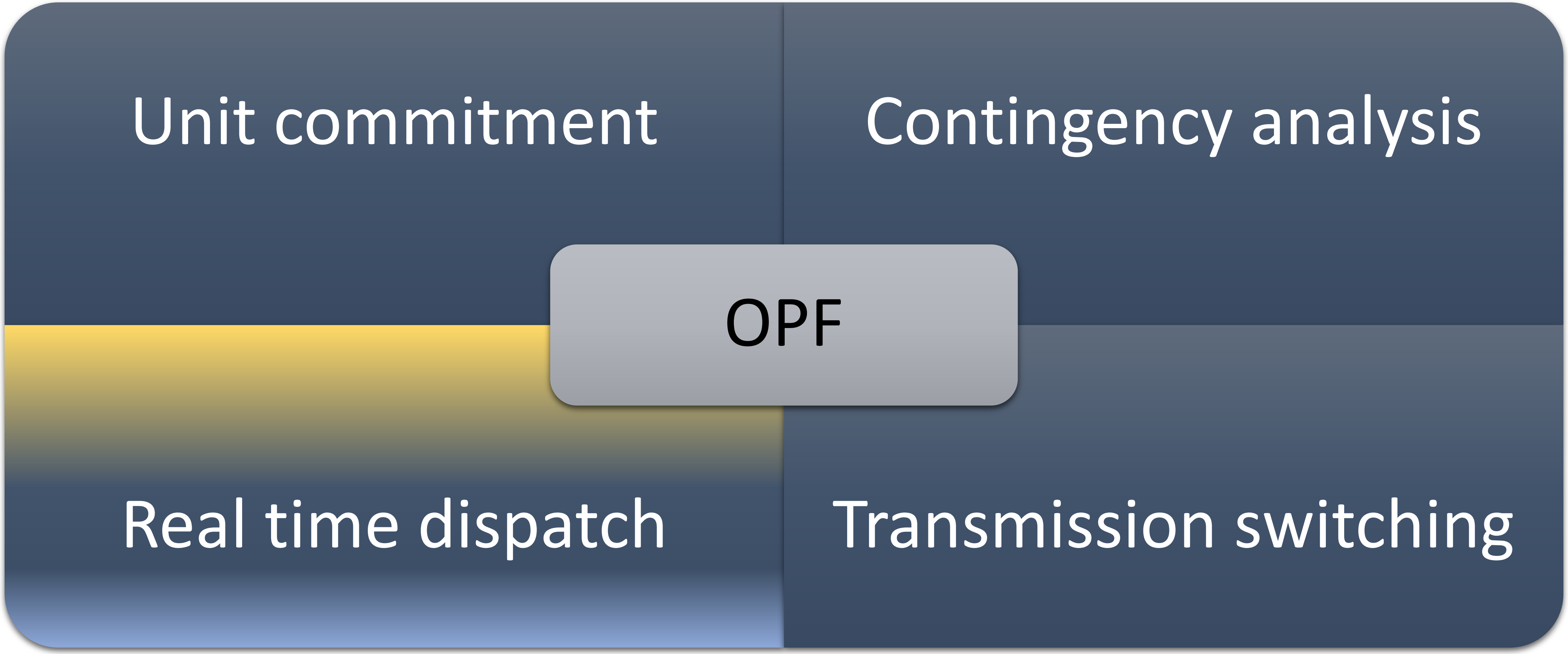
Optimal Power Flow underlies many market applications



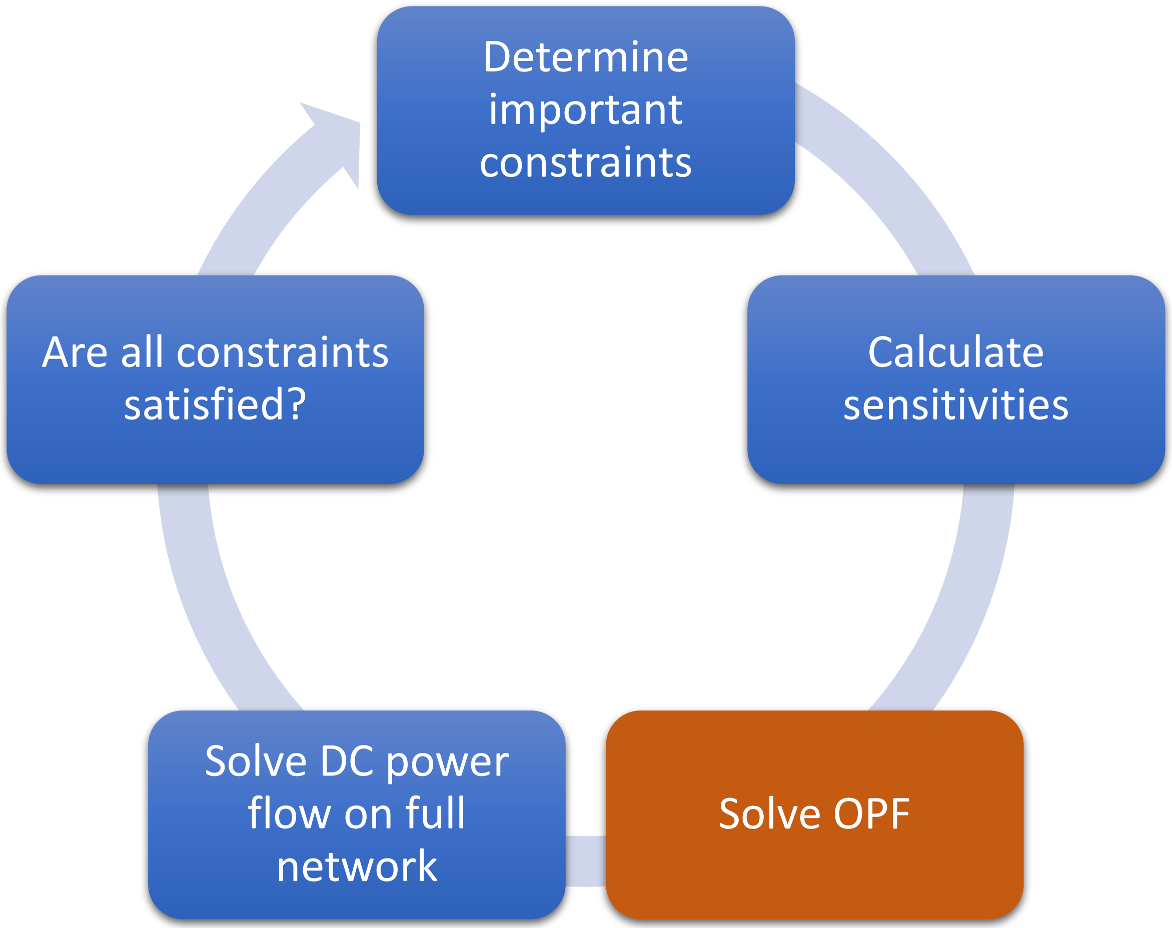
Comparison of AC and DC approaches

AC OPF-based models	**DC OPF-based models**

• Physics is correct
• Nonlinear, nonconvex
• Uses real and reactive power
• Relaxation (SDR,SOCR,QCR) to convex program
    • Promising approach, *but…*
    • Poor scaling in large systems
    • Nonlinear
    • Unsuitable for many applications
	• Physics is approximated
	• Linear, convex
	• Only considers real power
	• B-theta and distribution factor (PTDF) formulations
	    • B-theta common in academic literature
	    • ***PTDF used in ISO applications***

Optimal power flow is not solved in isolation

- Consider the "outer loop"
- PTDF advantages:
  - Linear
  - Compact formulation
  - Nonbinding constraints can be dropped
- AC OPF and B-theta require the full *explicit* formulations
  - Huge model size for AC OPF and AC OPF relaxations (SDP, etc.)



Research Questions for Linear OPF

How accurate can linear OPF models be?

What is the computational "cost" of more accurate OPF approximations?

What kind of simplifications can improve computational performance of dense OPF formulations?

What is the impact on solution accuracy?

Dense Optimal Power Flow: Distribution factor (PTDF) derivation

- Standard DC power flow assumptions result in:

$$\boldsymbol{p^f} = -\boldsymbol{BA\theta}$$

- Substitution into linear power balance constraints:

$$\boldsymbol{p^d} - \boldsymbol{p^g} - \boldsymbol{A}^\top \boldsymbol{BA\theta} = \boldsymbol{0}$$

- Define the PTDF as:

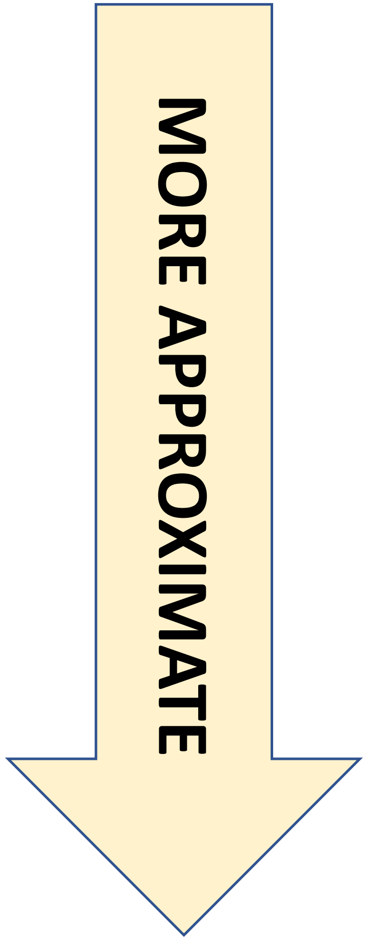
$$\boldsymbol{PTDF} \coloneqq -\boldsymbol{BA}(\boldsymbol{A}^\top \boldsymbol{BA})^{-1} \Rightarrow \quad \boldsymbol{p^f} = \boldsymbol{PTDF}(\boldsymbol{p^d} - \boldsymbol{p^g})$$

Issue: the B-theta power flow constraint matrix $-\boldsymbol{BA}$ is *sparse*, so inverting and calculating $\boldsymbol{PTDF}$ results in a *dense* constraint matrix.

Notation:

Power flow:	$\boldsymbol{p^f} \in \mathbb{R}^K$
Power demand:	$\boldsymbol{p^d} \in \mathbb{R}^N$
Power generation:	$\boldsymbol{p^g} \in \mathbb{R}^N$
Voltage angle:	$\boldsymbol{\theta} \in \mathbb{R}^N$
Adjacency matrix:	$\boldsymbol{A} \in \mathbb{R}^{K \times N}$
Susceptance mat.:	$\boldsymbol{B} \in \mathbb{R}^{K \times K}$

Four proposed models: Getting more accuracy out of linear OPF



Sparse (S-LOPF)

- First-order Taylor series approximation of decoupled AC power flow
- Mid-line power flow (Garcia et al., 2019)

Dense (D-LOPF)

- Dense factorization of the S-LOPF

Condensed (C-LOPF)

- Summation of branch losses to a system-wide loss variable
- Loss distribution factors to approximate effect of losses on branch flows

Real power (P-LOPF)

- Assume fixed voltages and reactive power flow
- Basically AC-linearized DC OPF

Sparse Linear OPF model (S-LOPF)

$$\min \sum_i C_i(p_i^g), \qquad s.t.:$$

$$\boldsymbol{p}^d - \boldsymbol{p}^g + \boldsymbol{A}^\top \boldsymbol{p}^f + \frac{1}{2}|\boldsymbol{A}|^\top \boldsymbol{p}^\ell = \boldsymbol{0}$$

$$\boldsymbol{q}^d - \boldsymbol{q}^g + \boldsymbol{A}^\top \boldsymbol{q}^f + \frac{1}{2}|\boldsymbol{A}|^\top \boldsymbol{q}^\ell = \boldsymbol{0}$$

$$\boldsymbol{p}^f = \boldsymbol{F}\boldsymbol{\theta} + \boldsymbol{F}^0$$

$$\boldsymbol{q}^f = \boldsymbol{G}\boldsymbol{v} + \boldsymbol{G}^0$$

$$\boldsymbol{p}^\ell = \boldsymbol{L}\boldsymbol{\theta} + \boldsymbol{L}^0$$

$$\boldsymbol{q}^\ell = \boldsymbol{K}\boldsymbol{v} + \boldsymbol{K}^0$$

$$(\boldsymbol{p}^f, \boldsymbol{q}^f, \boldsymbol{p}^\ell, \boldsymbol{q}^\ell, \boldsymbol{v}) \in \mathcal{T}$$

$$(\boldsymbol{p}^g, \boldsymbol{q}^g) \in \mathcal{G}$$

- Cost minimization
- Linear balance constraints
- First-order Taylor series expansion of power flow and branch losses
  - $\boldsymbol{F}, \boldsymbol{G}, \boldsymbol{L}, \boldsymbol{K}$ and offsets are calculated from the AC power flow equations
  - Analogous to $\boldsymbol{PTDF}$
- Transmission constraints $\mathcal{T}$
  - Piece-wise linear apparent power limits
  - Upper/lower voltage magnitude limits
- Generator constraints $\mathcal{G}$

Dense Linear OPF model (D-LOPF)

$$\min \sum_i C_i(p_i^g), \qquad s.t.:$$

$$\mathbf{1}^\top(\boldsymbol{p}^d - \boldsymbol{p}^g) + \mathbf{1}^\top \boldsymbol{p}^\ell = \mathbf{0}$$

$$\boldsymbol{p}^f = \widehat{\boldsymbol{F}}(\boldsymbol{p}^d - \boldsymbol{p}^g) + \widehat{\boldsymbol{F}}^0$$

$$\boldsymbol{q}^f = \widehat{\boldsymbol{G}}(\boldsymbol{q}^d - \boldsymbol{q}^g) + \widehat{\boldsymbol{G}}^0$$

$$\boldsymbol{p}^\ell = \widehat{\boldsymbol{L}}(\boldsymbol{p}^d - \boldsymbol{p}^g) + \widehat{\boldsymbol{L}}^0$$

$$\boldsymbol{q}^\ell = \widehat{\boldsymbol{K}}(\boldsymbol{q}^d - \boldsymbol{q}^g) + \widehat{\boldsymbol{K}}^0$$

$$\boldsymbol{v} = \widehat{\boldsymbol{S}}(\boldsymbol{q}^d - \boldsymbol{q}^g) + \widehat{\boldsymbol{S}}^0$$

$$(\boldsymbol{p}^f, \boldsymbol{q}^f, \boldsymbol{p}^\ell, \boldsymbol{q}^\ell, \boldsymbol{v}) \in \mathcal{T}$$

$$(\boldsymbol{p}^g, \boldsymbol{q}^g) \in \mathcal{G}$$

- Cost minimization
- System real power balance
  - Reactive balance is unnecessary
- Redefine $\boldsymbol{p}^f, \boldsymbol{q}^f, \boldsymbol{p}^\ell, \boldsymbol{q}^\ell$, and $\boldsymbol{v}$ using PTDF-like distribution factors

$$\widehat{\boldsymbol{F}} := -\boldsymbol{F}\left(\boldsymbol{A}^\top \boldsymbol{F} + \frac{\mathbf{1}}{\mathbf{2}}|\boldsymbol{A}|^\top \boldsymbol{L}\right)^{-1}$$

- Other constraints stay the same

Condensed Linear OPF model (C-LOPF)

$$\min \sum_i C_i(p_i^g), \qquad s.t.:$$

$$\mathbf{1}^\top(\boldsymbol{p}^d - \boldsymbol{p}^g) + p^\ell = \mathbf{0}$$

$$\boldsymbol{p}^f = \widehat{\boldsymbol{F}}(\boldsymbol{p}^d - \boldsymbol{p}^g) + \widehat{\boldsymbol{F}}^0$$

$$\boldsymbol{q}^f = \widehat{\boldsymbol{G}}(\boldsymbol{q}^d - \boldsymbol{q}^g) + \widehat{\boldsymbol{G}}^0$$

$$p^\ell = \check{\boldsymbol{L}}(\boldsymbol{p}^d - \boldsymbol{p}^g) + \check{\boldsymbol{L}}^0$$

$$q^\ell = \check{\boldsymbol{K}}(\boldsymbol{q}^d - \boldsymbol{q}^g) + \check{\boldsymbol{K}}^0$$

$$\boldsymbol{v} = \widehat{\boldsymbol{S}}(\boldsymbol{q}^d - \boldsymbol{q}^g) + \widehat{\boldsymbol{S}}^0$$

$$(\boldsymbol{p}^f, \boldsymbol{q}^f, p^\ell, q^\ell, \boldsymbol{v}) \in \mathcal{T}$$

$$(\boldsymbol{p}^g, \boldsymbol{q}^g) \in \mathcal{G}$$

- Cost minimization
- System power balance with system power losses
- Reduce number of constraints by summing $\boldsymbol{p}^\ell$ and $\boldsymbol{q}^\ell$ constraints

  $$\check{\boldsymbol{L}} := \mathbf{1}^\top \widehat{\boldsymbol{L}}, \qquad \check{\boldsymbol{L}}^0 := \mathbf{1}^\top \widehat{\boldsymbol{L}}^0$$
- Approximate branch losses with loss distribution factor in $\mathcal{T}$ constraints
- Generator constraints stay the same

Real Power Linear OPF model (P-LOPF)

$$\min \sum_i C_i(p_i^g), \qquad s.t.:$$

$$\mathbf{1}^\top(\boldsymbol{p}^d - \boldsymbol{p}^g) + p^\ell = \mathbf{0}$$

$$\boldsymbol{p}^f = \widehat{\boldsymbol{F}}(\boldsymbol{p}^d - \boldsymbol{p}^g) + \widehat{\boldsymbol{F}}^{\mathbf{0}}$$

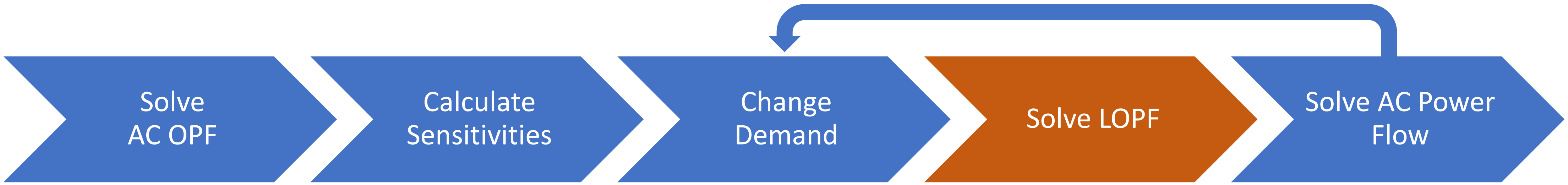
$$p^\ell = \breve{\boldsymbol{L}}(\boldsymbol{p}^d - \boldsymbol{p}^g) + \breve{\boldsymbol{L}}^{\mathbf{0}}$$

$$(\boldsymbol{p}^f, \boldsymbol{q}^f, p^\ell, q^\ell, \boldsymbol{v}) \in \mathcal{T}$$

$$(\boldsymbol{p}^g, \boldsymbol{q}^g) \in \mathcal{G}$$

- Cost minimization
- System power balance with system power losses
- Remove reactive power and voltage magnitude constraints
- Fix reactive power and voltage variables in $\mathcal{T}$ constraints
- Generator constraints stay the same

Methodology



Solve AC OPF
- Basepoint solution
- Solved using IPOPT

Calculate Sensitivities
- Real/Reactive power flow and line losses
- Voltage magnitude
- Sparse and dense calculations

Change Demand
- Uniform multiplier from 0.9 to 1.1
- 0.01 increments

Solve LOPF
- S/D/C/P-LOPF
- B-theta/PTDF
- Solved using Gurobi

Solve AC Power Flow
- Calculate power flow errors and violations
- Simulates “actual” dispatch from OPF solution

Extensive set of test cases:

PRELIMINARY results use full methodology to case588_sdet, nominal demand only to case6515_rte

'pglib_opf_case3_lmbd’,
'pglib_opf_case5_pjm’,
'pglib_opf_case14_ieee’,
'pglib_opf_case24_ieee_rts',
'pglib_opf_case30_as'
'pglib_opf_case30_fsr',
'pglib_opf_case30_ieee',
'pglib_opf_case39_epri',
'pglib_opf_case57_ieee',
'pglib_opf_case73_ieee_rts',
'pglib_opf_case89_pegase',
'pglib_opf_case118_ieee',
'pglib_opf_case162_ieee_dtc',
'pglib_opf_case179_goc',
'pglib_opf_case200_tamu',
'pglib_opf_case240_pserc',
'pglib_opf_case300_ieee',
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'pglib_opf_case588_sdet',
'pglib_opf_case1354_pegase',
'pglib_opf_case1888_rte',
'pglib_opf_case1951_rte',
'pglib_opf_case2000_tamu',
'pglib_opf_case2316_sdet',
'pglib_opf_case2383wp_k',
'pglib_opf_case2736sp_k',
'pglib_opf_case2737sop_k',
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'pglib_opf_case2746wp_k',
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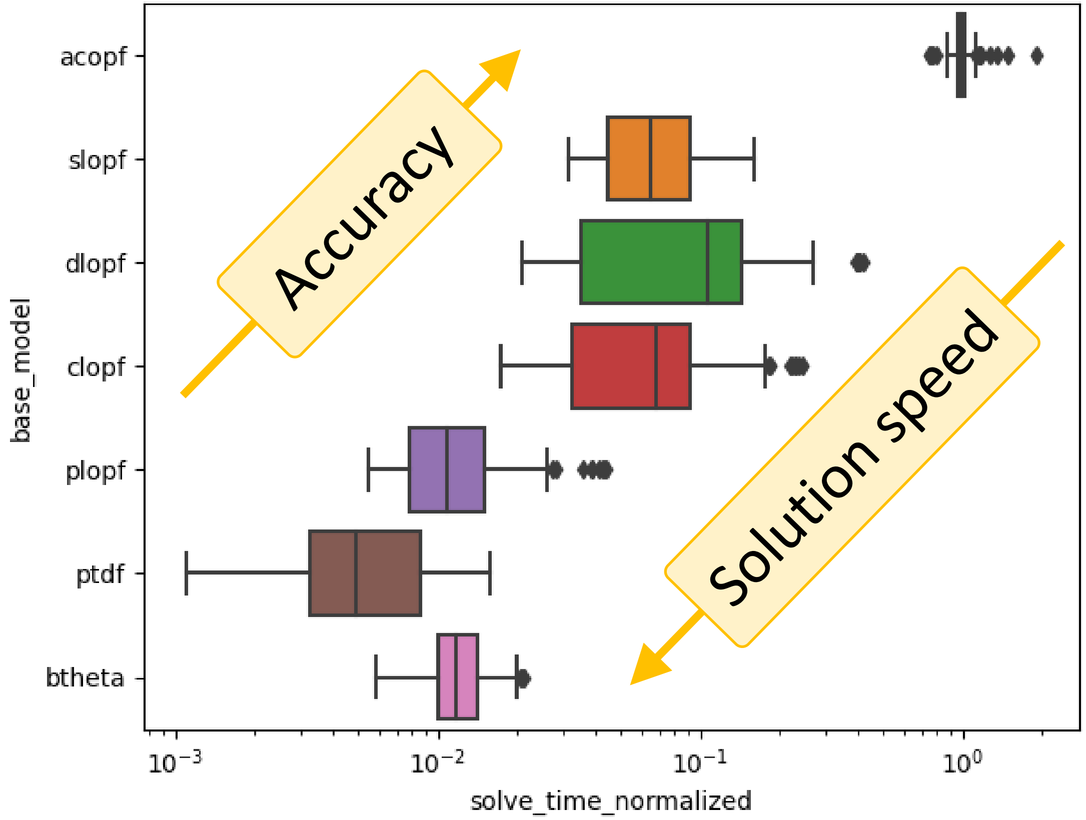
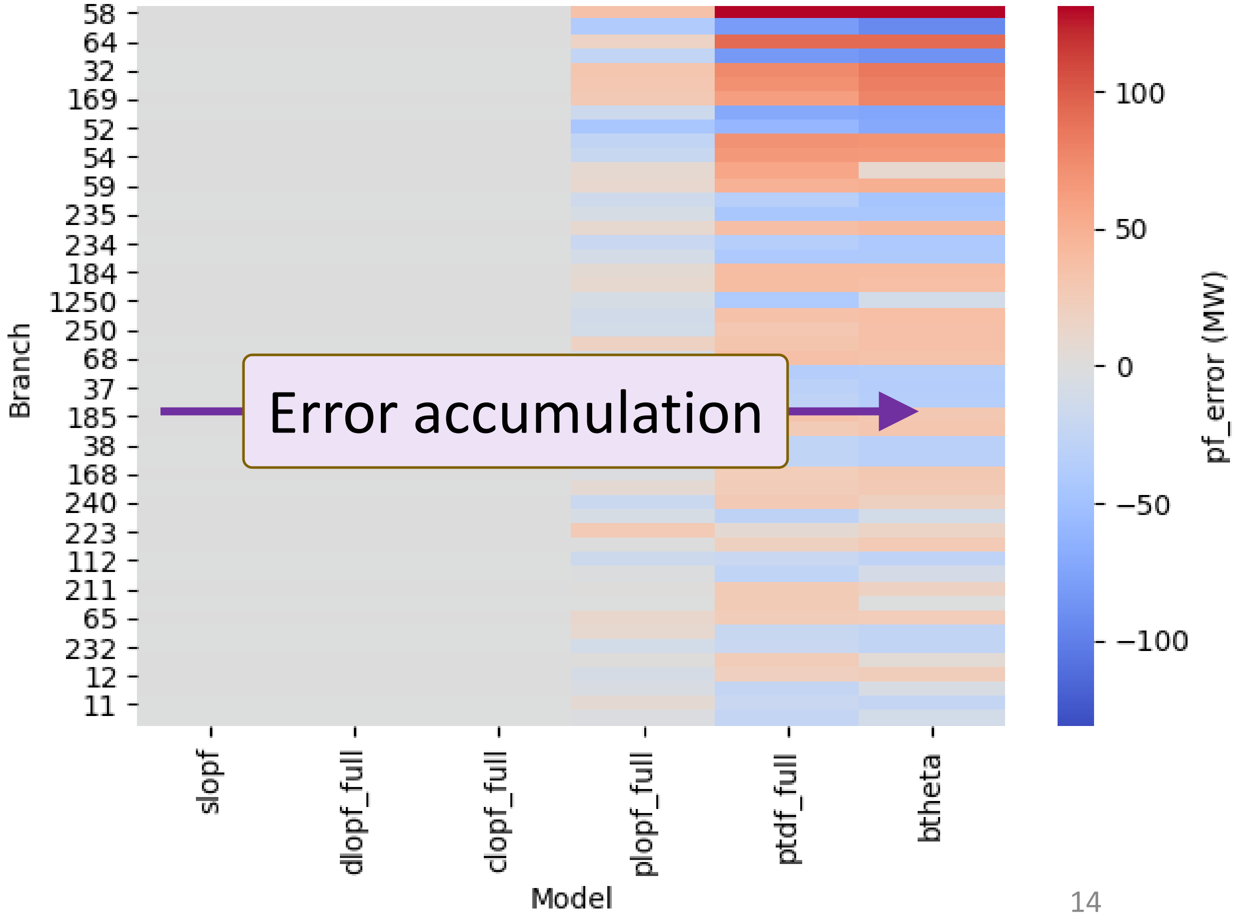
'pglib_opf_case2869_pegase',
'pglib_opf_case3012wp_k',
'pglib_opf_case3120sp_k',
'pglib_opf_case3375wp_k',
'pglib_opf_case4661_sdet',
'pglib_opf_case6468_rte',
'pglib_opf_case6470_rte',
'pglib_opf_case6495_rte',
'pglib_opf_case6515_rte',
'pglib_opf_case9241_pegase',
'pglib_opf_case10000_tamu',
'pglib_opf_case13659_pegase'

“Vanilla” Implementations: DC OPF / LOPF / AC OPF

Solution Speed - cases with 3-588 buses

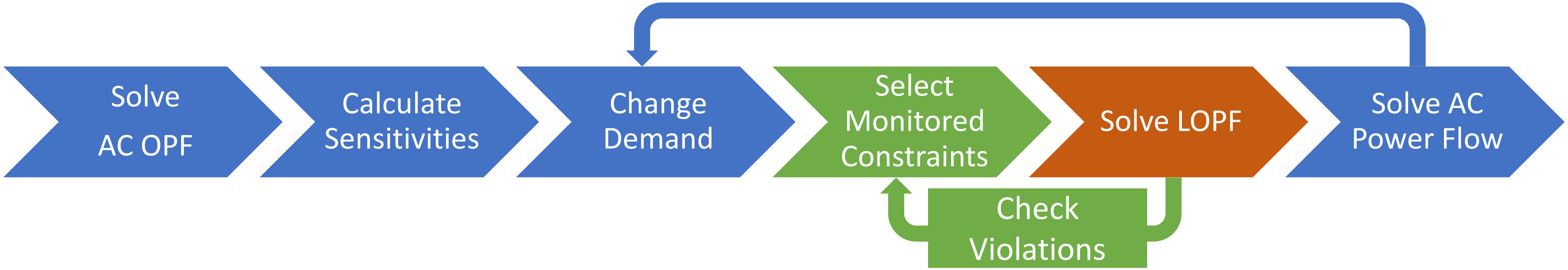


AC Power Flow Error - case2383wp_k



PRELIMINARY RESULTS

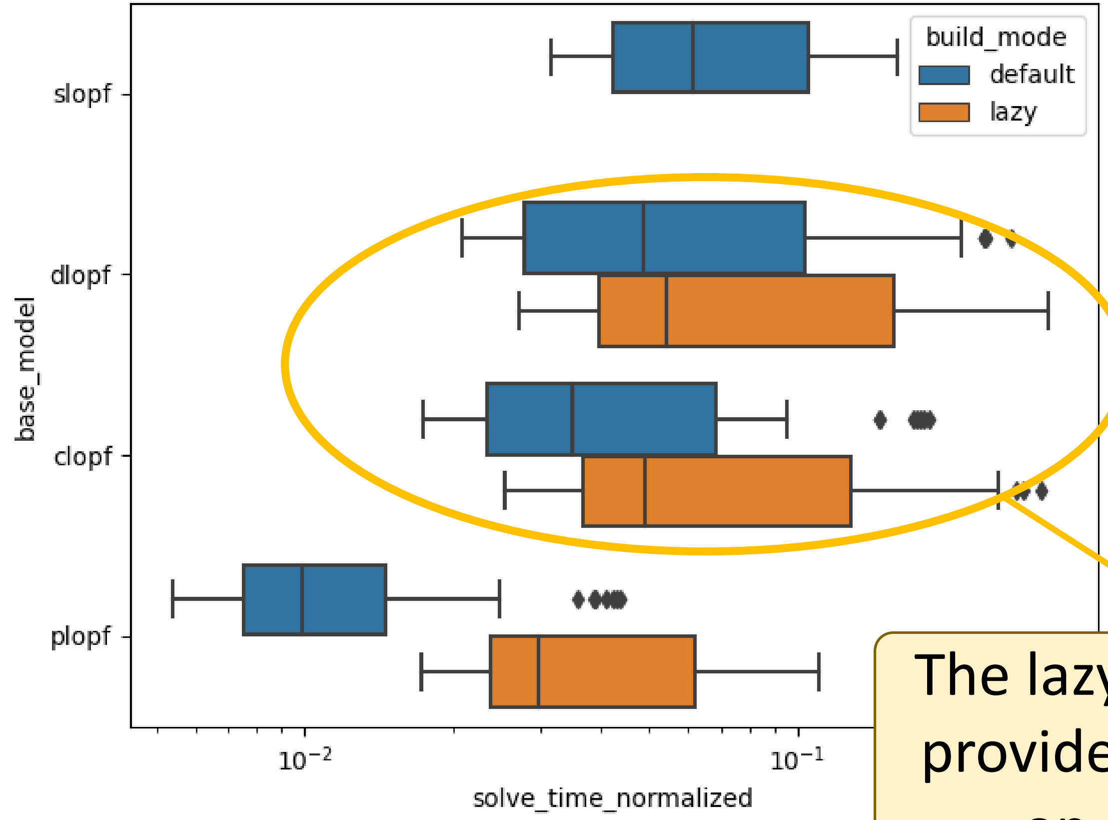
Methodology – “Lazy” Constraint Algorithm



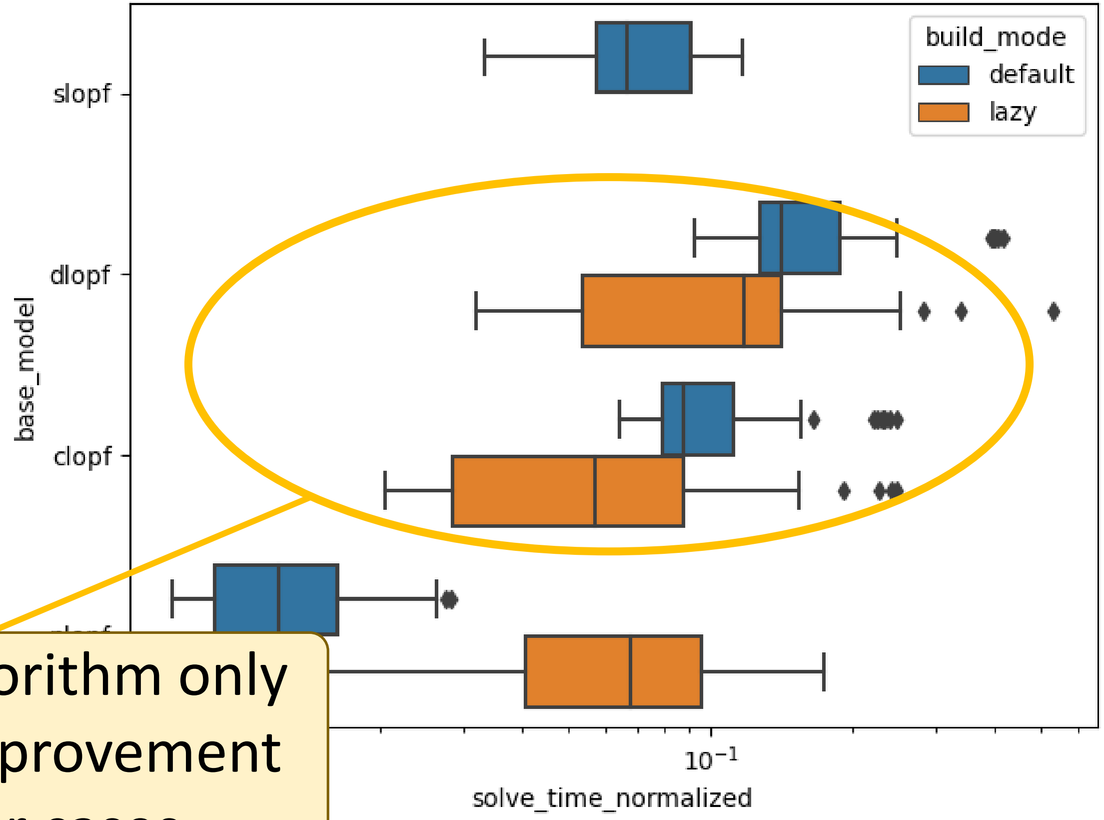
- The lazy constraint algorithm can be implemented on “dense” OPFs
  - Implemented with Gurobi’s persistent solver
  - D/C/P-LOPF and PTDF formulation of the DC OPF
  - Reduces transmission constraints to around 5% of full model
  - Reduces overall model size to about 50%
- Not possible on sparse (S-LOPF/B-theta) formulations
  - Removing line results in a new topology

Lazy algorithm benefits cases with >100 buses

Cases with 3-89 buses



Cases with 118-588 buses



The lazy algorithm only provides improvement on larger cases

PRELIMINARY RESULTS

Hybrid line loss constraints for D-LOPF

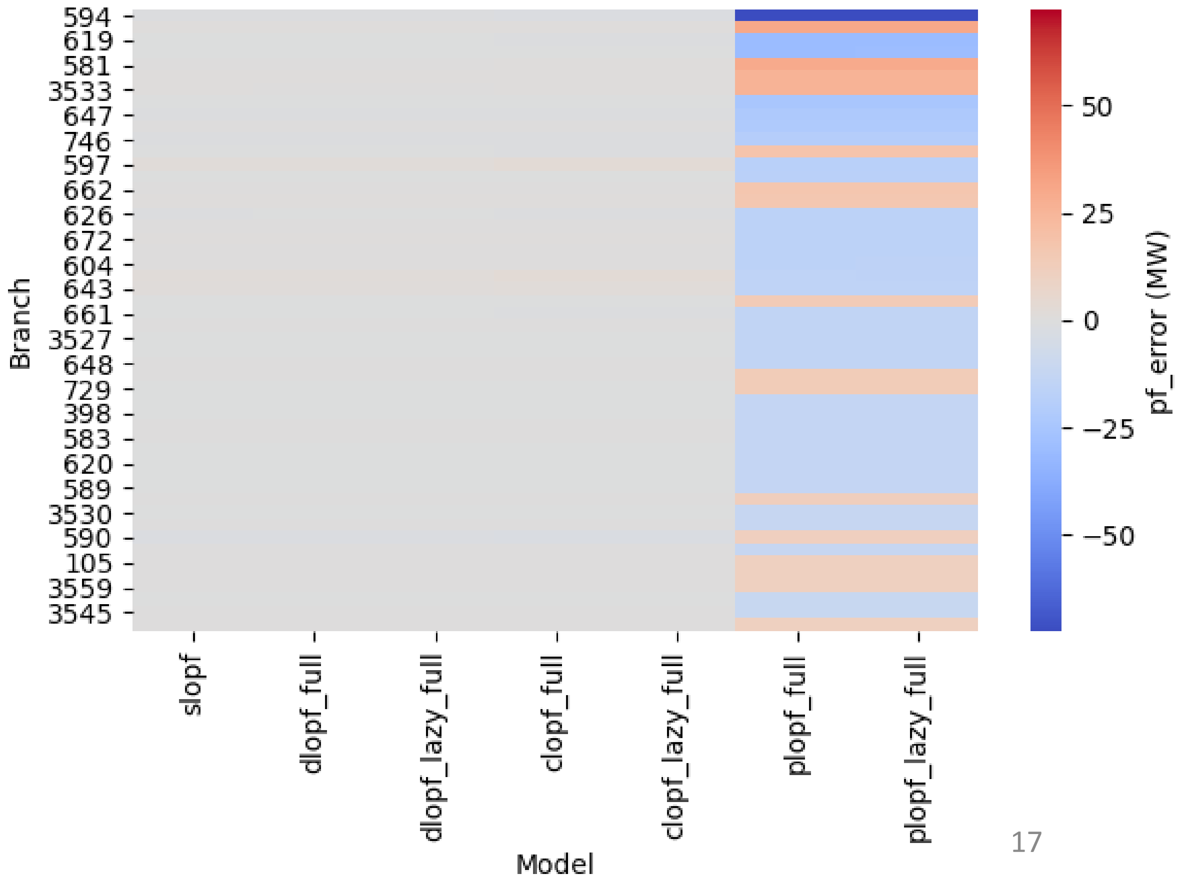
- Lazy algorithm does NOT allow us to ignore branch loss constraints

$$\sum_i (p_i^d - p_i^g) + \sum_{k \in \mathcal{K}'} p_k^\ell + \sum_{\boldsymbol{k \in \mathcal{K} \setminus \mathcal{K}'}} \boldsymbol{(p_k^\ell = 0)} = 0$$

- Solution: implement a hybrid model with residual losses

$$\sum_i (p_i^d - p_i^g) + \sum_{k \in \mathcal{K}'} p_k^\ell + \boldsymbol{p^{resid}} = 0$$

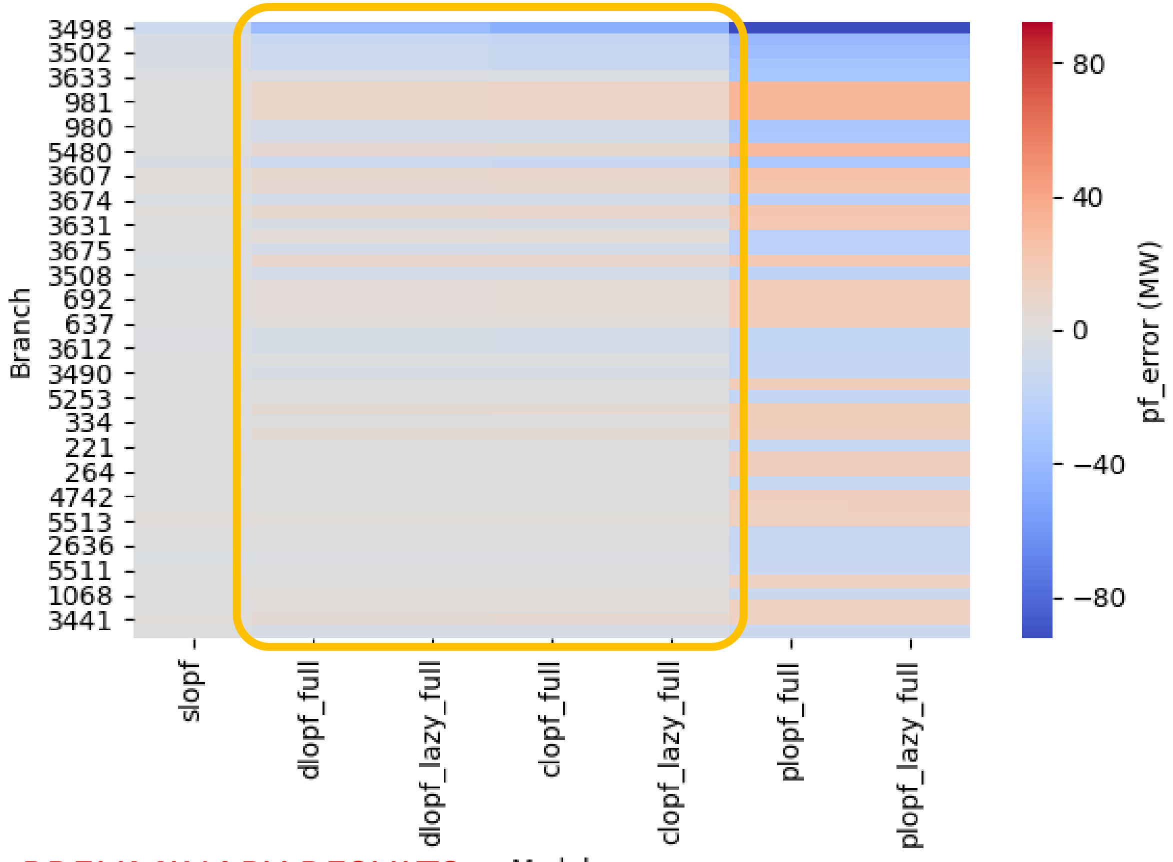
Result is similar to C-LOPF formulation (case3375wp_k)



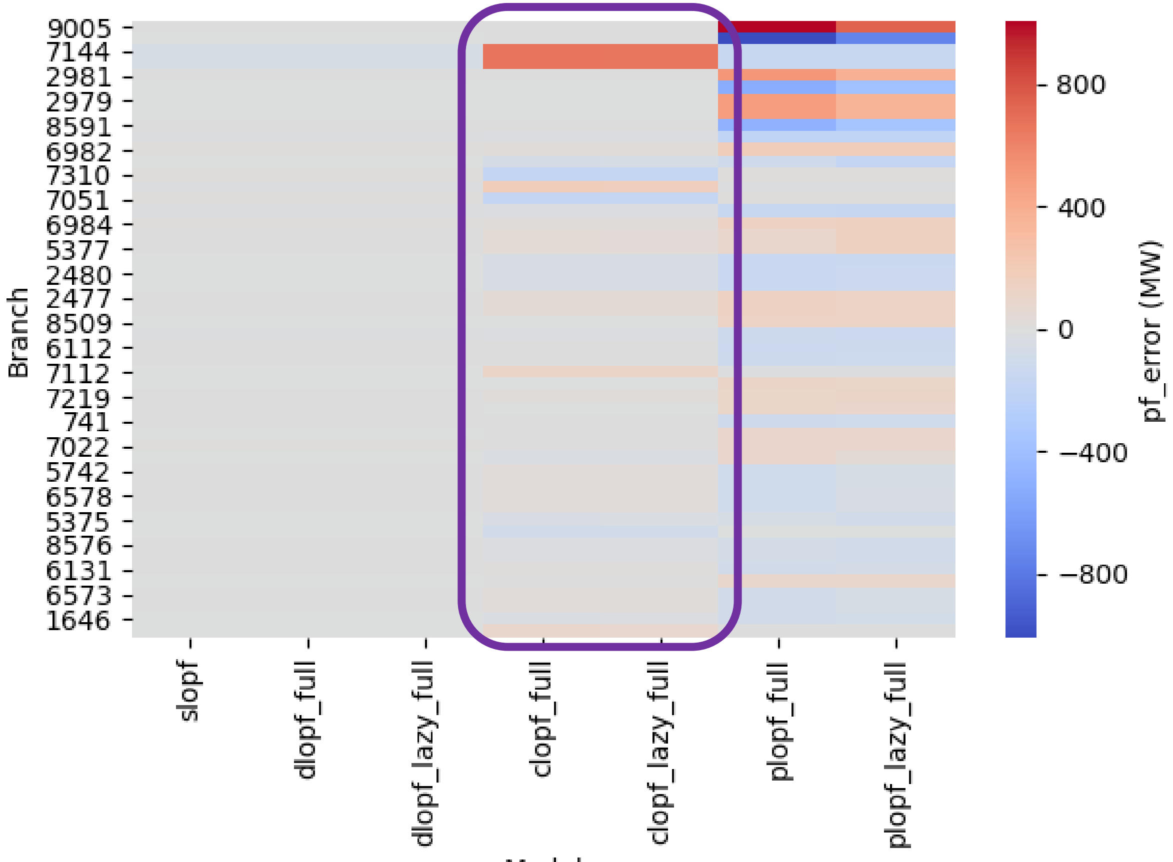
PRELIMINARY RESULTS

Loss summation in C-LOPF and hybrid D-LOPF can cause accumulation of power flow errors

Hybrid/C-LOPF error accumulation (case4661_sdet)



C-LOPF error accumulation (case6495_rte)



PRELIMINARY RESULTS

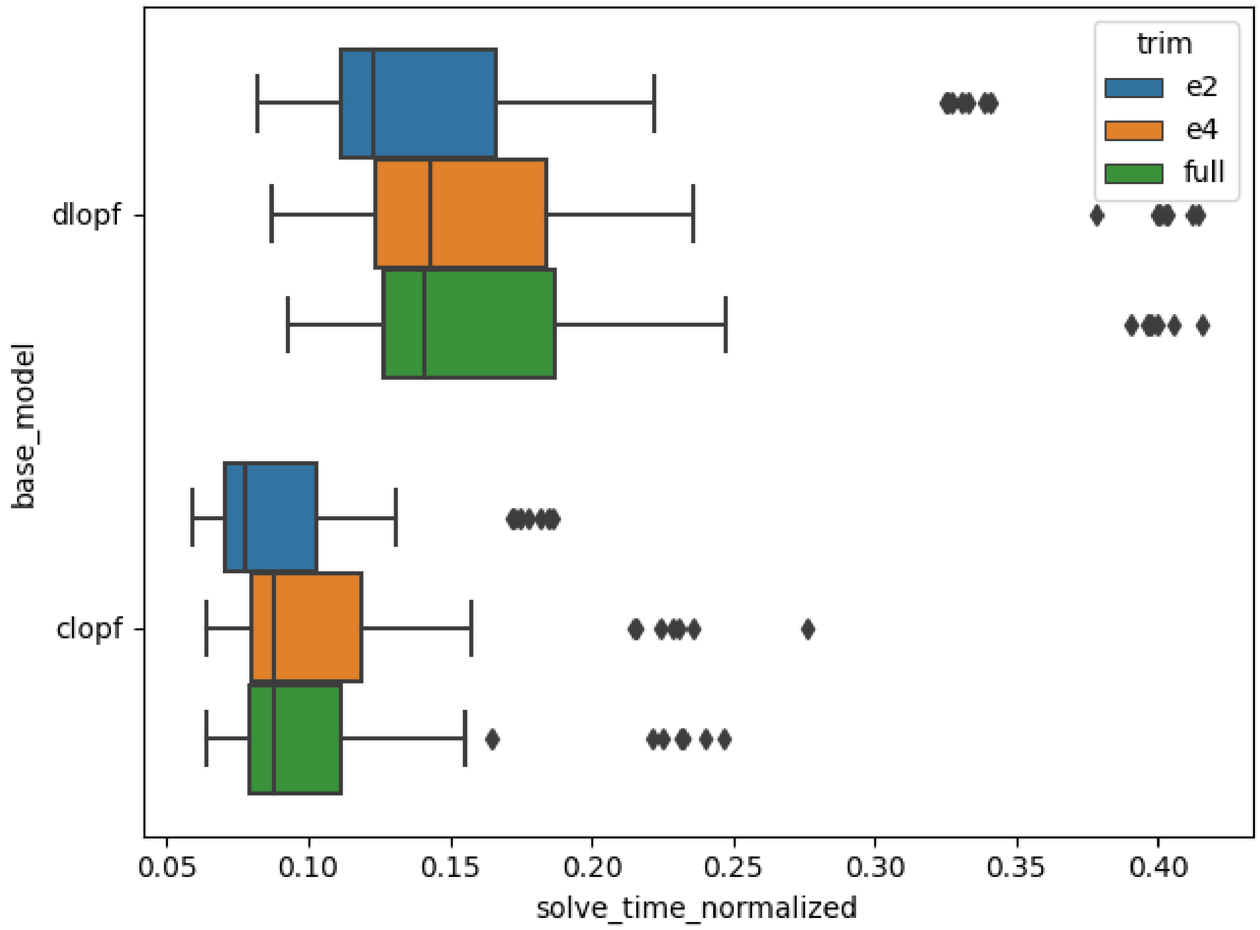
Factor truncation improves constraint sparsity but can also accumulate power flow errors

- Dense models can be made more "sparse" by eliminating small coefficients

$$\hat{F}_{ik}^{\varepsilon} = \hat{F}_{ik} 1_{\{\hat{F}_{ik} < \varepsilon\}}$$

$$\hat{F}_{ik}^{0,\varepsilon} = \hat{F}_{ik}^{0} + \sum_{\{i : \hat{F}_{ik} \geq \varepsilon\}} \hat{F}_{ik} \left(p_i^d - p_i^g\right)$$

- Errors are usually small



PRELIMINARY RESULTS

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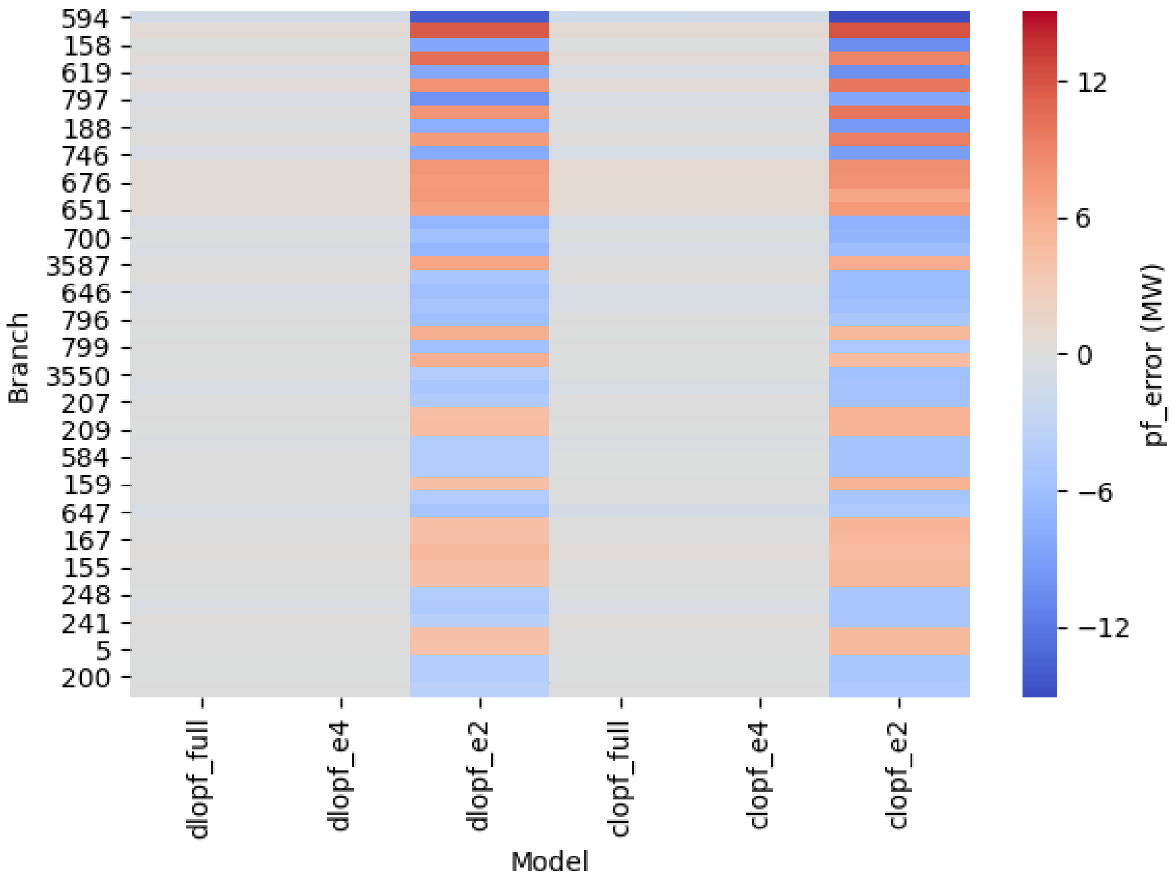
- Errors are ***usually*** small

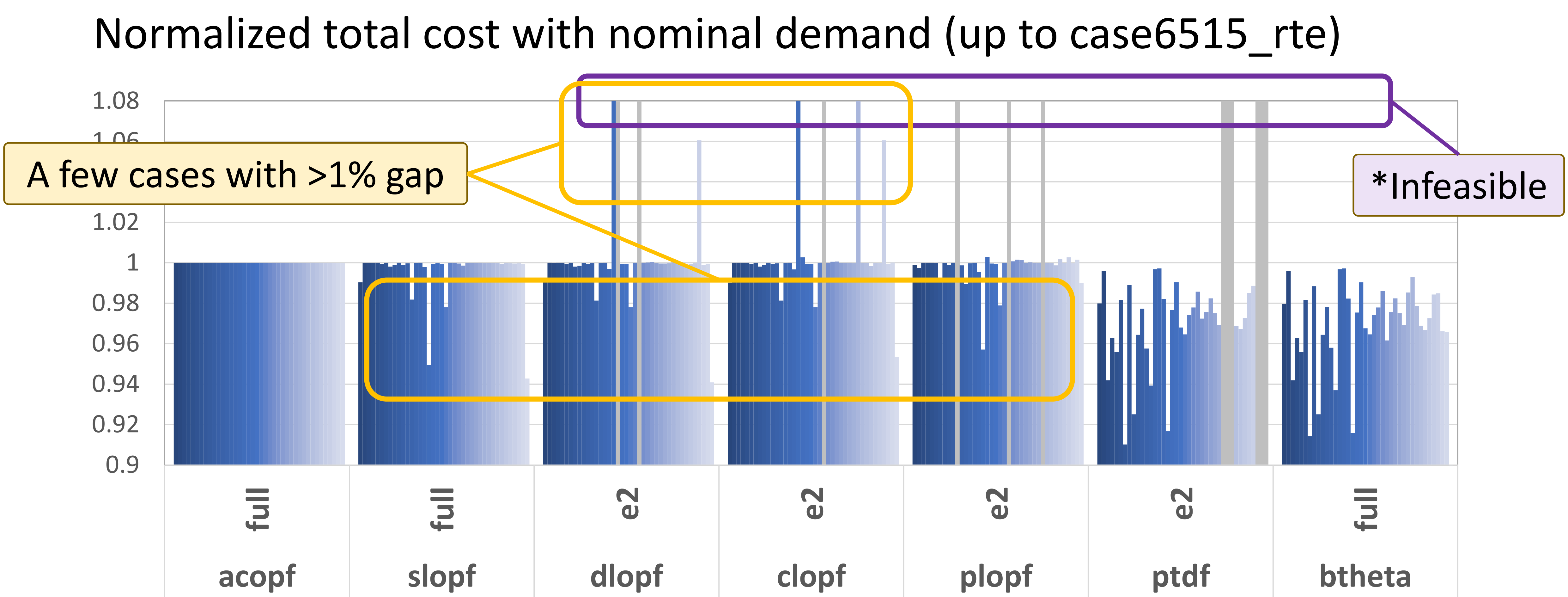
Distortions in case3375wp_k



PRELIMINARY RESULTS

Conclusions

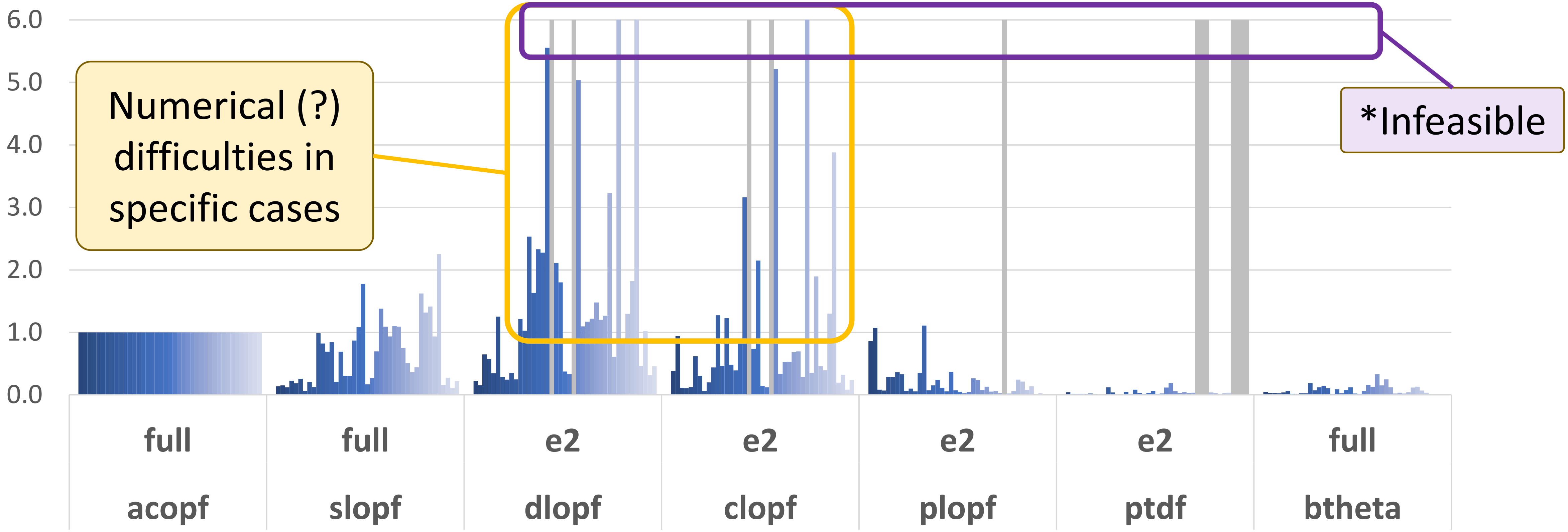
How accurate can LOPF results be?



PRELIMINARY RESULTS

What is the computational “cost”?

Normalized solution time at nominal demand (up to case6515_rte)



PRELIMINARY RESULTS

Conclusions

- Benefits of AC-linearized optimal power flow models
  - vs DC OPF: power flow accuracy
  - vs AC OPF: computational speed
  - Based on use of state estimator data in real-world applications
- Simplifications required for good “PTDF” implementations
  - Active set algorithms
  - Line loss relaxation (system/residual losses)
  - Factor truncation
  - Not trivial… Lots of trial and error
- Future work:
  - Preprocessing memory & speed improvements
  - More intelligent active set implementation
  - Computational performance in unit commitment models

Questions