A UNIFIED APPROACH TO SOLVE CONVEX HULL PRICING AND AVERAGE INCREMENTAL COST PRICING

FERC Technical Conference on Increasing Real-Time and Day-Ahead Market Efficiency through Improved Software
June 23-25, 2020

Yonghong Chen, Consulting Advisor, R&D, MISO

Richard O'Neill, ARPA-E, US Department of Energy (Washington, DC)

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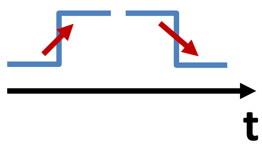
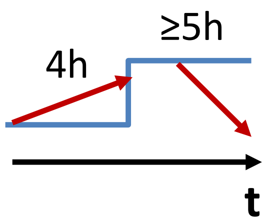
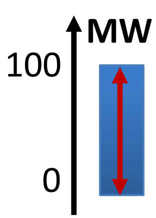
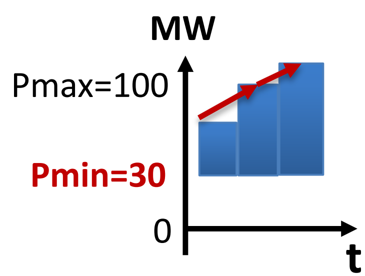
Non-convexity creates challenges for pricing

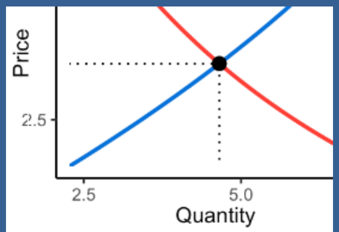
Ideal	***Reality***
 *Can start up or shut down* ***any time*** *when needed*	*Requires startup and notification times and has minimum run / down times* 
 *Can be dispatched within the capacity with only* ***Variable cost*** ***($/MWh)***	*Has minimum MW and ramp rate limits* ***Variable cost and Fixed cost*****: startup cost ($/start), no load cost ($/h)*** 

Scheduling

- Merit order based on variable cost

Pricing uses Locational Marginal Price (LMP)

- Marginal variable cost to serve last MW



Scheduling (difficult)

- Security constrained unit commitment

Pricing (difficult)

- How to allocate fixed cost to the right time intervals and average over the right MW ranges

Market clearing and pricing models

- Unit commitment and economic dispatch (UCED) model for market clearing:

$$v(d) = min \quad \sum_{g \in G} C_g(p_g, u_g) \qquad (1)$$
$$s.t. \quad \sum_{g \in G} p_g = \mathrm{d} \qquad (2)$$
$$(p_g, u_g) \in \mathrm{X}_g \quad \forall g \in G \qquad (3)$$
$$\mathrm{X}_g \subseteq R_+^T \times \{0,1\}^T$$

  – Optimal solution: (p_g^*, u_g^*) for $g \in G$

- Market clearing price (LMP and MCP)
  – Fixing commit variables and solve LP
  – Locational marginal price, π, is calculated from the dual variables

Make-whole payment

- LMP only reflects the marginal cost. It may not be able to cover the total avoidable cost under the UCED solution.
- Define the profit under commitment block $B_{g,j}$ as:

$$\varphi_{g,B_{g,j}}(\pi, p^*, u^*) = \sum_{t \in B_{g,j}} [\pi_t p^*_{gt} - C_{gt}(p^*_{gt}, u^*_{gt})]$$

- Make-whole payment (MWP) is used to compensate for profit loss and is defined as:

$$M_g(\pi, p^*_g, u^*_g) = \max\{0, -\sum_{j=1}^{k_g} \varphi_{g,B_{g,j}}(\pi, p^*_g, u^*_g)\}$$

- All ISO/RTOs compensate for MWP
  - However, MWP is not transparent

Out-of-money generators and uplift

- Profit for a generator under π' and commitment / dispatch signal

$$\varphi_g(\pi', p_g^*, u_g^*) = \pi' p_g^* - C_g(p_g^*, u_g^*)$$

- Given price π', generator owners may solve profit maximizing problem

$$\omega_g(\pi') = max_{g \in G}\, [\pi' p_g - C_g(p_g, u_g)\,|(p_g, u_g) \in X_g]$$

- A generator is "out-of-money" if uplift

$$U_g(\pi', p^*, u^*) = \omega_g(\pi') - [\pi' p_g^* - C_g(p_g^*, u_g^*)] > 0$$

Non-convexity or sub-optimal commitment

- Uplift includes MWP and Lost Opportunity Cost (LOC)
  - MWP: when resource can make more profit by generating less MW than RTO's instruction.
  - LOC: when resource may make more profit by generating more MW than RTO's instruction

Convex Hull Pricing (CHP) Pricing[1]

CHP pricing is proposed in non-convex market to minimize total uplift and to better support the market clearing solution. CHP incorporates both the marginal costs and the avoidable fixed operating costs of available resources in the market.

It considers the total available capacity of the online and offline resources and balances the incentives to follow RTOs' instructions for all resources. It may still require MWP and/or LOC under CHP.

CHP price can be solved through the Lagrangian dual problem. However, it's difficult to converge. Recent advance on UCED resource formulation allows it to be solved with LP relaxation of the UCED problem.

Current implementations are mostly single interval approximation for fast start resources

[1] P. Gribik, W. Hogan, and S. Pope, "Market-clearing electricity prices and energy uplift," Harvard Univ., Cambridge, MA, working paper, 2007.

CHP and Lagranian Dual Problem[1]

- The Lagrangian dual function after dualizing power balance equation is

$$q(\pi, d) = \sum_{g \in G} min \ \{C_g(p_g, u_g) - \pi' p_g \mid (p_g, u_g) \in X_g]\} \ + \pi' d$$

Profit maximization of individual resources

- The Lagrangian dual problem is to find the dual maximizer price. π^* for $Q(d) = max_{\pi} \ \ q(\pi, d)$
- Under any price π, the gap between the UCED problem and its dual is exactly the total uplift.
  - $D(d, \pi) = v(d) - \ q(\pi, d) = \sum_{g \in G} U_g(\pi, p^*, u^*)$
- The solution of the Lagrangian dual problem can minimize the uplift and is the convex hull price.
- $Q(d)$ is very difficult to converge.

[1] P. Gribik, W. Hogan, and S. Pope, "Market-clearing electricity prices and energy uplift," Harvard Univ., Cambridge, MA, working paper, 2007.

Solve CHP with LP Relaxation (LIP) of UCED[2][3]

UCED LP relaxation (LIP):

$$v\_rel(d) = min \sum_{g \in G} C_g^{**}(p_g, u_g)$$

$$s.t \sum_{g \in G} p_g = d$$

$$(p_g, u_g) \in conv(X_g)$$

$$0 \leq u_g \leq 1 \quad \forall g \in G$$

CHP problem MIP:

$$L = Max_\pi q(\pi, d)$$

$$s.t. q(\pi, d) = \sum_{g \in G} min \ \{C_g(p_g, u_g) - \pi' p_g \mid (p_g, u_g) \in X_g]\} + \pi' d$$

$$L\_Relax = Max_\pi q(\pi)$$

$$s.t. \quad q(\pi) = \sum_{g \in G} min\{C_g^{**}(p_g, u_g) - \pi' p_g \mid (p_g, u_g) \in conv(X_g), 0 \leq u_g \leq 1]\} + \pi' d$$

Convex envelope

Convex hull

- Under the convex hull and convex envelope formulation: CHP can be solved with LP relaxation (LIP) of UCED[2]
- An extended integral UC formulation is developed and an iterative algorithms is developed in [3] to solve CHP with multiple LIPs.

[2] B. Hua and R. Baldick, "A convex primal formulation for convex hull pricing," IEEE Transactions on Power Systems, 2017

[3] Y. Yu, Y. Guan, Y. Chen, "An Extended Integral Unit Commitment Formulation and An Iterative Algorithm for Convex Hull Pricing", IEEE Transactions on Power Systems, 2020

Average Incremental Cost (AIC) Pricing[4][5]

AIC pricing is proposed in non-convex market as the rough equivalent to marginal cost pricing in convex markets. It may serve as an entry signal in addition to the LMP.

Similar to CHP, the AIC pricing mechanism produces prices that incorporate both the marginal costs and the avoidable fixed operating costs of a dispatched resource.

AIC pricing focuses on the dispatched MW of online resources and eliminates MWP.

AIC pricing can be solved similar to CHP – By adjusting resource upper bounds based on the UCED solution, the LP relaxation of the UCED problem can also be used to solve AIC prices.

[4] R. P. O'Neill, A. Castillo, B. Eldridge and R. B. Hytowitz, "Dual Pricing Algorithm in ISO Markets," IEEE Transactions on Power Systems, 2017.
[5] Richard O'Neill, Notes on AIC pricing, 2019

Solve AIC with LP Relaxation (LIP) of UCED[6]

AIC can also be solved with LP Relaxation

- By adding **p-cut** based on optimal UCED solution (p_g^*, u_g^*),$g \in G$ under the convex hull and convex envelope formulation.

1. Define set S^{MWP} for commitment blocks requiring MWP under LMP:

- $S^{MWP} = \{(g,t) |\, p_{gt}^* > 0, t \in B_{g,j}, \varphi_{g,B_{g,j}}\left(LMP, p_g^*, u_g^*\right) < 0\}$

2. Define p-cut: cut off uncommitted and un-dispatched regions

- $X_{p_{g,\varepsilon}^*}^{AIC} = \{(p_g, u_g) \in \mathrm{X}_g, p_{gt} \leq \ p_{gt}^{AIC-max}),\ \forall g \in G, t \in T\}$,
- where $p_{gt}^{AIC-max} =$
- $\begin{cases} \min(p_{gt}^* + \varepsilon, p_{gt}^{max}) & if\ (g,t) \in S^{MWP} \\ 0 & if\ p_{gt}^* = 0 \\ p_{gt}^{max} & if\ p_{gt}^* > 0, and\ (g,t) \notin S^{MWP} \end{cases}$

[6] Y. Chen, R. P. O'Neill, P. Whitman, "A Unified Approach to Solve Convex Hull Pricing and Average Incremental Cost Pricing with Large System Study", IEEE Transactions on Power Systems, Under Review

Unified approach to solve CHP and AIC

UCED MIP with AIC p-cut:

$$v^{AIC}(d, p^*_{gt}, \varepsilon) = \min \sum_{g\in G} C_g(p_g, u_g)$$

$$s.t. \sum_{g\in G} p_g = d$$

$$(p_g, u_g) \in X^{AIC}_{p^*_{g,\varepsilon}}, \forall g \in G$$

$$X_g \subseteq R^T_+ \times \{0,1\}^T$$

(p^*_g, u^*_g) *is also an optimal solution*

AIC problem MIP

$$L = Max_\pi q(\pi, d)$$

$$s.t. q(\pi, d) = \sum_{g\in G} \min \{C_g(p_g, u_g) - \pi' p_g \mid (p_g, u_g) \in X^{AIC}_{p^*_{g,\varepsilon}}]\} + \pi' d$$

UCED LP relaxation (LIP)

$$v\_rel^{AIC}(d, p^*_{gt}, \varepsilon) = \min \sum_{g\in G} C^{**}_g(p_g, u_g)$$

$$s.t \sum_{g\in G} p_g = d$$

$$(p_g, u_g) \in conv\left(X^{AIC}_{p^*_{g,\varepsilon}}\right)$$

$$0 \leq u_g \leq 1 \quad \forall g \in G$$

$X^{AIC}_{p^*_{g,\varepsilon}}$ is a sub-region of X_g with added p-cuts based on p^*_g

$$L\_Relax = Max_\pi q(\pi)$$

$$s.t. \quad q(\pi) = \sum_{g\in G} \min \{C^{**}_g(p_g, u_g) - \pi' p_g \mid (p_g, u_g) \in conv\left(X^{AIC}_{p^*_{g,\varepsilon}}\right), 0 \leq u_g \leq 1]\} + \pi' d$$

Convex envelope

Convex hull

Under the convex hull and convex envelope formulation, AIC can be solved with LP relaxation of UCED within a sub-region defined based on UCED solution

AIC price solved from LP relaxation with p-cut can eliminate make whole payments

- **Proposition 1**: There is no opportunity cost for $v^{AIC}(d, p_{gt}^*, \varepsilon)$ under LMP. The total uplift equals the total MWP.
- **Proposition 2**: Under optimal solution (p_g^*, u_g^*) , For the commitment block requiring MWP under LMP, i.e., the subset $(g, t) \in S^{MWP}$, the solution in $v\_rel^{AIC}(d, p_{gt}^*, \varepsilon)$ cannot be $u_{g,t}^{**} = 0$.
- **Proposition 3**: Under pricing solution π^{**} from $v\_rel^{AIC}(d, p_{gt}^*, \varepsilon)$, the commitment block under subset $(g, t) \in S^{MWP}$ has \$0 MWP when $\varepsilon \to 0$.

AIC can be solved with LP relaxation under the following conditions:

1) Add **p-cut** based on the **optimal** UCED solution
2) Apply the extended individual resource convex hull & convex envelope formulation in UCED

2-Generator Example

Using the 3-binary formulation and solving AIC with LP relaxation with p-cut, the price can eliminate MWP but, it is higher than necessary and not a good entry signal.

			Cost			Ramp Rate		
	Pmin	Pmax	$/MWh	Startup	NoLoad	Normal	Startup	Shut down
Gen1	0	100	10	0	0	100	100	100
Gen2	20	35	50	1000	30	5	22.5	35

3-binary formulation:

$$\min \sum_{t=1}^{3} 10 \cdot p_{1,t} + \sum_{t=1}^{3} (30 \cdot u_{2,t} + 50 \cdot p_{2,t} + 1000 \cdot v_{2,t})$$

Limit constraints:

$$0 \leq p_{1,t} \leq 100 \quad for \ 1 \leq t \leq 3 \quad (a1)$$

$$20u_{2,t} \leq p_{2,t} \leq 35\, u_{2,t} \quad for \ 1 \leq t \leq 3 \quad (a2)$$

Ramping constraints:

$$p_{2,t} - p_{2,t-1} \leq 5u_{2,t-1} + 22.5v_{2,t} \quad for \ 1 \leq t \leq 3 \quad (a3)$$

$$p_{2,t-1} - p_{2,t} \leq 5u_{2,t} + 35e_{2,t} \quad for \ 2 \leq t \leq 3 \quad (a4)$$

Binary constraints:

$$u_{2,t} - u_{2,t-1} = v_{2,t} - e_{2,t} \quad for\ 1 \leq t \leq 3$$

$$with \ u_{2,0} = 0 \ for\ initially\ off \quad (a5)$$

$$v_{2,t} \leq u_{2,t} \quad for\ 1 \leq t \leq 3 \quad (a6)$$

$$v_{2,t} \leq 1 - u_{2,t-1} \quad for\ 1 \leq t \leq 3 \quad (a7)$$

Power balance constraint:

$$p_{1,t} + p_{2,t} = LD_t \quad for\ 1 \leq t \leq 3 \quad (a8)$$

$$v_{2,t}, u_{2,t}, e_{2,t} \ are\ binary \quad for\ 1 \leq t \leq 3 \quad (a9)$$

u,v,e: commitment, startup, shutdown variables

p: dispatch variable

t	1	2	3	
LD	95	100	130	
LMP	10	10	90	
p2	20	25	30	sum
Gen2 Profit	-$1,830	-$1,030	$1,170	-$1,690
Total profit	$6,310	Max profit		$8,000
Uplift	$1,690	MWP		$1,690

t	1	2	3	
LD	95	100	130	
AIC2	10	10	1,161	sum
Gen2 Profit	-$1,830	-$1,030	$33,305	$30,445
Total Profit	$145,563	Max profit		$148,141
Uplift	$2,578	MWP		$0

Using extended generator convex hull / convex envelope formulation and solving AIC with LP relaxation with p-cut: The price is just enough to eliminate MWP and a good entry signal.

Extended generator CHP / convex envelope formulation

$$\min \sum_{t=1}^{3} 10 \cdot p_{1,t} + 1000 \cdot \left(\sum_{tk\in\{02,03,13\}} y_{2,tk} + \sum_{t\in\{1,2,3\}} w_{2,t}\right) + 30 \cdot \left(\sum_{t\in\{0,1,2\}} \sum_{s\in[t+1,3]} w_{2,t} + \sum_{tk\in\{02,03,13\}} \sum_{s\in[t+1,k-1]} y_{2,tk}\right) + 50 \cdot \left(\sum_{t\in\{0,1,2\}} \sum_{s\in[t+1,3]} qw_{2,t}^{s} + \sum_{tk\in\{02,03,13\}} \sum_{s\in[t+1,k-1]} qy_{2,tk}^{s}\right)$$

Limit constraints

$0 \le p_{1,t} \le 100 \qquad for\ 1 \le t \le 3 \qquad$ (b1)

$20 w_{2,t} \le qw_{2,t}^{s} \le 35\, w_{2,t} \quad t \in [0,2], s \in [t+1,3] \qquad$ (b2)

$20 y_{2,tk} \le qy_{2,tk}^{s} \le 35\, y_{2,tk}$

$tk \in \{02,03,13\}, s \in [t+1, k-1] \qquad$ (b3)

Ramping constraints

$qy_{2,tk}^{t+1} \le 22.5\, y_{2,tk},\ \ qw_{2,t}^{t+1} \le 22.5\, w_{2,t} \qquad$ (b4)

$qy_{2,03}^{2} - qy_{2,03}^{1} \le 5\, y_{2,03},\ qy_{2,03}^{1} - qy_{2,03}^{2} \le 5\, y_{2,03},$

$qw_{2,t}^{s+1} - qw_{2,t}^{s} \le 5\, w_{2,t},\ t \in [0,2], s \in [t+1,3]$

$qw_{2,t}^{s} - qw_{2,t}^{s+1} \le 5\, w_{2,t},\ t \in [0,2], s \in [t+1,3]$

Binary constraints

$-o_{2,0} + y_{2,02} + y_{2,03} + w_{2,0} = 0, \quad -o_{2,1} + y_{2,13} + w_{2,1} = 0,$

$-o_{2,2} + w_{2,2} = 0, \quad y_{2,02} - z_{2,22} - z_{2,23} = 0,$

$y_{2,03} + y_{2,13} - z_{2,33} = 0, \quad o_{2,0} + o_{2,1} + o_{2,2} \le 1$

t	1	2	3	
LD	95	100	130	
AIC	10	10	146.33	sum
Gen2 Profit	-$1,830	-$1,030	$2,860	-$0.10
Total Profit	$13,633	Max profit		$14,770
Uplift	$1,138	MWP		$0.10

The final dispatch MW of Gen2:

$p_{2,1} = qy_{2,02}^{1} + qy_{2,03}^{1} + qw_{2,0}^{1}$

$p_{2,2} = qy_{2,03}^{2} + qy_{2,13}^{2} + qw_{2,0}^{2} + qw_{2,1}^{2}$

$p_{2,3} = qw_{2,0}^{3} + qw_{2,1}^{3} + qw_{2,2}^{3}$

Power balance constraint:

$p_{1,t} + p_{2,t} = LD_t \qquad for\ 1 \le t \le 3 \qquad$ (a8)

$o_{2,t}$: stay off through t and start up at the beginning of t+1, for t=0,1,2

$w_{2,t}$:start up at the beginning of t+1 and stay on until the end, for t=0,1,2.

$y_{2,tk}$: start up at the beginning of t+1 and shut down at the beginning of k, for $tk \in \{02,03,13\}$.

$z_{2,tk}$: shut down at the beginning of t and stay off until the beginning of k+1, for $tk \in \{22,23,33\}$.

CHP and AIC prototype study on MISO size cases

CHP and AIC are prototyped to study revised MISO DA cases including

- Energy only with Transmission constraints
- Most of generation constraints (limit, ramping, min run / min down / max run times)
- Generation costs: hot / intermediate / cold startup times and costs, no load cost and piece wise linear incremental energy cost

Three versions of AIC:

- "AIC": with p-cut and extended generator convex hull / convex envelope formulation
- "AIC2": with p-cut and 3-binary formulation
- "AIC34": with additional p-cut, binary cuts, flow cuts and 3-binary formulation[7]
  - Binary cuts: $u \leq u^*, v \leq v^*, \mathrm{e} \leq e^*$

Solving Time (s)							
SCUC time	305	2277	3809	1533	2566	368	6777
LMP	negligible						
CHP	1208	6671	7552	5329	10500	4832	5675
AIC	344	581	337	405	456	387	344
AIC2	153	149	184	91	124	90	98
AIC34	98	103	118	101	129	103	102

*Intel Haswell processor @ 2.5 GHz, 512GB RAM, 32 sockets per CPU, 1 core per socket, 1 thread per core

[7] R. P. O'Neill, Y. Chen, "The One-Pass AIC Approach with Multi-Step Marginal Costs, Ramp Constraints and Reserves", Working Paper, 2020.

CHP and AIC prototype study on MISO DA case *(cont.)*

- MWP, uplift and profit

MWP (percentage relative to LMP MWP)							
LMP	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
CHP	18.23%	11.36%	15.76%	26.55%	15.57%	1.64%	6.34%
AIC	0.00%	0.02%	0.05%	0.16%	0.02%	0.22%	0.00%
AIC2	0.00%	0.01%	0.05%	0.16%	0.02%	0.20%	0.00%
AIC34	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Gen Uplift (percentage relative to LMP Gen profit)							
LMP	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
CHP	9.00%	15.28%	9.72%	5.67%	15.90%	6.20%	5.65%
AIC	106.85%	102.80%	131.72%	79.16%	111.12%	72.32%	90.75%
AIC2	108.29%	112.72%	138.28%	83.33%	113.45%	74.83%	91.08%
AIC34	104.17%	101.58%	139.98%	92.88%	111.50%	94.62%	102.51%
FTR Uplift (percentage relative to LMP Gen profit)							
LMP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CHP	1.97%	4.01%	5.76%	1.19%	4.59%	1.92%	3.56%
AIC	0.20%	1.02%	3.34%	0.85%	1.18%	1.20%	2.60%
AIC2	0.20%	1.06%	3.31%	0.85%	1.22%	1.19%	2.59%
AIC34	0.03%	0.19%	3.92%	0.41%	0.03%	0.32%	1.09%
Gen Profit (percentage relative to LMP Gen profit)							
LMP	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
CHP	89.51%	104.13%	100.11%	102.37%	104.42%	104.57%	101.33%
AIC	105.44%	110.18%	108.24%	108.56%	112.71%	107.52%	111.57%
AIC2	105.48%	111.26%	109.50%	108.46%	113.61%	107.73%	113.21%
AIC34	104.61%	107.02%	111.37%	107.21%	111.64%	106.63%	110.57%

AIC: zero or close to zero MWP

- Small residuals on AIC and AIC2 are due to small MIP gap in UCED solution
- AIC34 can eliminate MWP under sub-optimal UCED solution by adding additional p-cuts, binary cuts and flow cuts [7].

CHP: Minimum uplift

Transmission constraints not binding in UCED may bind in CHP and/or AIC runs, resulting in FTR uplift

Application of the advanced pricing methods

Remaining technical issues to be addressed before CHP or AIC can be used as market clearing prices

- Real time rolling market clearing window issue is not fully addressed: commitment costs prior to the clearing window become sunk costs
- Emerging non-traditional resources: on-going research to derive extended convex hull & convex envelope formulation for multi-configuration combined cycle, storage, etc.

Potential near term application: estimate the total operational cost for reserve and regional transfer constraints

- Under uncertainty, operations may take emergency actions for system-wide or regional reliability
- Shadow prices from current operating reserve and regional transfer constraints may not fully reflect the cost of those actions
  - MISO single interval approximate ELMP may help to reflect fast-start resource commitment cost
- AIC and CHP prices from multi-interval UCED can better reflect full costs from commitment and emergency actions
  - Potentially help on defining reserve demand curves (e.g., ORDC) and regional transfer demand curves

References

[1] P. Gribik, W. Hogan, and S. Pope, “Market-clearing electricity prices and energy uplift,” Harvard Univ., Cambridge, MA, USA, working paper, 2007.

[2] B. Hua and R. Baldick, “A convex primal formulation for convex hull pricing,” IEEE Transactions on Power Systems, vol. 32, no. 5, pp. 3814–3823, 2017

[3] Y. Yu, Y. Guan, Y. Chen, “An Extended Integral Unit Commitment Formulation and An Iterative Algorithm for Convex Hull Pricing”, IEEE Transactions on Power Systems, Accepted, 2020

[4] R. P. O’Neill, A. Castillo, B. Eldridge and R. B. Hytowitz, "Dual Pricing Algorithm in ISO Markets," IEEE Transactions on Power Systems, vol. 32, no. 4, pp. 3301-3310, 2017.

[5] Richard O’Neill, Notes on AIC pricing, 2019

[6] Y. Chen, R. P. O’Neill, P. Whitman, “A Unified Approach to Solve Convex Hull Pricing and Average Incremental Cost Pricing with Large System Study”, IEEE Transactions on Power Systems, Under Review

[7] R. P. O’Neill, Y. Chen, “The One-Pass AIC Approach with Multi-Step Marginal Costs, Ramp Constraints and Reserves”, Working Paper, April, 2020.