Strengthened MILP Formulation for the Edge-based Combined-Cycle Unit Model

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Outline

1 Introduction
   - Introduction
   - Motivation

2 Edge-Based Formulation
   - Transition Graph Representation
   - Operating Constraints

3 Strengthened Edge-Based Formulation
   - Tighter Constraints
   - Strong Valid Inequalities

4 Case Studies

5 Conclusion
Combined-Cycle Units

- Combustion Turbines: use natural gas, produce electricity and heat.
- Heat Recovery Steam Generator: produce steam.
- Steam Turbines: use steam to produce electricity.
Current Practice

- Aggregated modeling approach [1]:
  - Treats the whole combined-cycle unit as a traditional thermal unit.
  - Less decision variables.
  - Cannot reflect the relationship between CT and ST.

- Pseudo unit approach [2]:
  - Associates each combustion turbine (CT) with a portion of the steam turbine (ST).
  - Less decision variables.
  - Cannot capture the transition process.

- Configuration-based model [3],[4],[5],[6]:
  - Represents each combination of CTs and STs as a configuration.
  - Cannot capture the operating constraints such as min-up/-down constraints for each turbine.
Configuration-Based Model

- Work at different typical configurations: 0CT + 0ST, 1CT + 0ST, 2CTs + 0ST, 1CT + 1ST, and 2CTs + 1ST
- Each configuration is treated as a pseudo unit: generation limits, ramping rates, and min-up/-down constraints.

Figure: Transition Graph for 2CTs + 1ST
Challenges

- Time $t$, Configuration 1 online
- Time $t + 2$ Load increases, Generation amount increases, ST starts up, Configuration 2 online
- Time $t + 3$ Works at Configuration 2 for several time periods (e.g., 4 time periods).
- If load increases dramatically, it might be more than the capacity of Configuration 2 at time $t + 3, t + 4$.
- Second CT can start up, if this CT satisfies its own min-down time requirement.

Figure: Transition Graph for 2CTs + 1ST
Challenges

- Improve flexibility?
- Design the min-up/-down constraints for each turbine instead of each configuration.

Figure: Transition Graph for 2CTs + 1ST
Motivation

- Improved the accuracy of the configuration-based model [7].

Method

- Proposed an edge-based formulation based on the transition graph.

Contribution

- Exactly described the physical constraints (in particular, min-up/-down restrictions for each turbine) and transition costs between different configurations.
- Increased the flexibility of the combined-cycle units in terms of unit commitment.
- Explored the structure of the state transition graph for combined-cycle units (such as the network flow structure) that commercial optimization solvers, e.g., CPLEX, can recognize.
Strengthened Edge-Based Combined-Cycle Unit Model

- **Motivation**
  - Reduce the computational time in the day-ahead unit commitment engine caused by combined-cycle units.

- **Method**
  - Cutting plane method.

- **Contribution**
  - Derived tighter min-up/-down and ramping rate constraints for a combined-cycle unit.
  - Provided several families of stronger valid inequalities of ramping rates for a combined-cycle unit by exploring the structure of the transition graph.
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Transition Graph

- Use complete transition graph (distinguish two CTs).
- Edge binary variables \( z^a_t \): transition action at each time period.

Unique constraints:

\[
\sum_{a \in A} z^a_t = 1, \forall t. \quad (1)
\]

Figure: Complete Transition Graph for 2CTs + 1ST
Network Flow

### Configuration Status

\[
\sum_{a \in (A_{in}^k \cup A_{sl}^k)} z_t^a
\]  
(2)

### Logical Constraints

\[
\sum_{a \in (A_{in}^k \cup A_{sl}^k)} z_t^a = \sum_{a \in (A_{out}^k \cup A_{sl}^k)} z_{t+1}^a, \forall k \in C, \forall t.
\]  
(3)

**Figure:** Edges of One Node
Min-up Time

- **Config 0**: CT1 + CT2
- **Config 1**: CT1
- **Config 2**: CT2
- **Config 3**: CT1 + ST
- **Config 4**: CT2 + ST
- **Config 5**: CT1 + CT2
- **Config 6**: CT1 + CT2 + ST

- **CT1 Starts up**: a01, a05, a25, a46

**Figure**: Start-up CT1
Min-up Time

- **CT1 starts up:** $a_{01}, a_{05}, a_{25}, a_{46}$
- **CT1 cannot shut down:** $a_{10}, a_{50}, a_{52}, a_{64}$

**Figure:** Start-up CT1
**Min-up Time**

- **CT1 starts up:** $a_{01}$, $a_{05}$, $a_{25}$, $a_{46}$
- **CT1 cannot shut down:** $a_{10}$, $a_{50}$, $a_{52}$, $a_{64}$
- **Configurations without CT1 cannot be online:** Config 0, Config 2, Config 4

**Figure:** Start-up CT1
Min-up Time

- CT1 starts up: a01, a05, a25, a46
- CT1 cannot shut down: a10, a50, a52, a64
- Configurations without CT1 cannot be online: Config 0, Config 2, Config 4
- Edges connected with Red configurations cannot be active.

Figure: Start-up CT1
Introduction

Edge-Based Formulation

Strengthened Edge-Based Formulation

Case Studies

Conclusion

**Min-up Time Constraints**

\[
\sum_{a \in \bigcup_{k \in C^i} A^a_{k}} z^a_T \leq 1 - \sum_{a \in A^a_{i}} z^a_t, \forall i \in U^{CT} \cup U^{ST},
\]

\[
\forall \tau \in \{t + 1, \cdots, \min\{T_{end}, T^i_{mu} + t - 1\}\}, \forall t.
\]
Min-down Time Constraints

\[
\sum_{a \in \bigcup_{k \in \mathcal{C}_i} \mathcal{A}_{i}^{\text{on}}} z_{a}^{\tau} \leq 1 - \sum_{a \in \mathcal{A}_{i}^{\text{sd}}} z_{a}^{i}, \forall i \in \mathcal{U}^{\text{CT}} \cup \mathcal{U}^{\text{ST}},
\]

\[
\tau \in \{t + 1, \ldots, \min\{T_{\text{end}}, T_{i}^{\text{md}} + t - 1\}\}, \forall t.
\]
Ramping Constraints

- If this particular edge is not active (i.e., $z^a_{t+1} = 0$), this ramping constraint is relaxed, following the definition of $P^{\text{cap}}$

- Otherwise, if this edge is active (i.e., $z^a_{t+1} = 1$), this edge provides the ramping limit for the whole combined-cycle unit, because only one of the edges can be active at each time period.

\[
p_{t+1} - p_t \leq RU_a z^a_{t+1} + P^{\text{cap}} (1 - z^a_{t+1}), \forall a \in A, \forall t, \tag{6}\]
\[
p_t - p_{t+1} \leq RD_a z^a_{t+1} + P^{\text{cap}} (1 - z^a_{t+1}), \forall a \in A, \forall t. \tag{7}\]
Reduced State Transition Graph

**Figure:** Reduced State Transition Graph for 2CT+1ST
Cutting Planes

Strong valid inequalities to cut off fractional solutions.
Cutting Planes

Strong valid inequalities to cut off fractional solutions.
Convex Hull

The smallest convex feasible region containing all feasible integer solutions
Min-up Time Constraints

- If turbine \( i \) is online at time period \( t \), then this turbine starts up at most once during time interval \([t - T^i_{mu} + 1, t - 1]\).
- If turbine \( i \) starts up at time interval \([t - T^i_{mu} + 1, t - 1]\), then the configurations without turbine \( i \) cannot be online at time period \( t \).

\[
\sum_{\kappa=1}^{T^i_{mu}-1} \sum_{a \in A^s_{i}^{su}} z^{a}_{t-\kappa} \leq 1 - \sum_{a \in \bigcup_{k \in C^i_{off}} A^a_{k}} z^{a}_{t}, \quad \forall i \in U^CT \cup U^{ST}, \forall t \in \{T^i_{mu}, \ldots, T_{end}\}. \tag{8}
\]
Min-down Time Constraints

- If one of the arcs in $A_{sd}^i$, representing the shut-down process of turbine $i$, is active during time interval $[t - T_{md}^i + 1, t - 1]$, then arcs $\bigcup_{k \in C_{on}^i} A_{all}^k$ connected to the configurations $(C_{on}^i)$ with turbine $i$ cannot be active.

- The configurations with turbine $i$ must be offline at time period $t$ when turbine $i$ shuts down at time interval $[t - T_{md}^i + 1, t - 1]$.

\[
T_{md}^i - 1 \sum_{\kappa=1}^{T_{md}^i-1} \sum_{a \in A_{sd}^i} z_{t-\kappa}^a \leq 1 - \sum_{a \in \bigcup_{k \in C_{on}^i} A_{all}^k} z_t^a,
\]

$\forall i \in U^{CT} \cup U^{ST}, \forall t \in \{T_{md}^i, \ldots, T_{end}\}$. (9)
Ramping Rate Constraints

- Since only one of the arcs in the transition graph can be active at each time period $t$, only one item in the right-hand side of (10) can be positive and all others would be zeros.

- The positive item represents the active arc that provides the ramping up rate limit. The same analysis can be applied to ramping down constraints (11).

\[
p_{t+1} - p_t \leq \sum_{a \in A} RU_a z_{t+1}^a, \forall t \in T, \tag{10}
\]

\[
p_t - p_{t+1} \leq \sum_{a \in A} RD_a z_{t+1}^a, \forall t \in T. \tag{11}
\]
Single-Arc Ramping Up Rate Inequalities

\[ p_t^m - p_t^n \leq RU^{a(n,m)}z_{t+1}^{a(n,m)} + \overline{P}_m \left( \sum_{a \in (A_{in}^m \cup A_{sl}^m)} z_{t+1}^{a} \right) \\
- \overline{P}_n \left( \sum_{a \in (A_{in}^n \cup A_{sl}^n)} z_{t}^{a} \right) + (\overline{P}_n - \overline{P}_m)z_{t+1}^{a(n,m)}, \]

\[ \forall a(n, m) \in A, \forall t \in T, \quad (12) \]

**Table: Validity of Ramping Up Inequalities (12)**

<table>
<thead>
<tr>
<th>Case</th>
<th>Value of Binary Variables</th>
<th>Inequality</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( \sum_{a \in (A_{in}^n \cup A_{sl}^n)} z_t^a )</td>
<td>( \sum_{a \in (A_{in}^m \cup A_{sl}^m)} z_{t+1}^a )</td>
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<tr>
<td>4</td>
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</tr>
</tbody>
</table>
Single-Arc Ramping Down Rate Inequalities

\[ p^n_t - p^m_{t+1} \leq \text{RD}^{a(n,m)} z^a_{t+1} + P_n \left( \sum_{a \in (A^\text{in}_n \cup A^\text{sl}_n)} z^a_t \right) \]

\[ - P_m \left( \sum_{a \in (A^\text{in}_m \cup A^\text{sl}_m)} z^a_{t+1} \right) + (P_m - P_n) z^a_{t+1}, \]

\[ \forall a(n, m) \in \mathcal{A}, \forall t \in T. \quad (13) \]

Table: Validity of Ramping Down Inequalities (13)

<table>
<thead>
<tr>
<th>Case</th>
<th>Value of Binary Variables</th>
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<td>$\sum_{a \in (A^\text{in}_n \cup A^\text{sl}_n)} z^a_t$</td>
<td>$\sum_{a \in (A^\text{in}<em>m \cup A^\text{sl}<em>m)} z^a</em>{t+1} \leq z^a</em>{t+1}$</td>
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Suppose that the combined-cycle unit works on Configuration $m$ at time period $t + 1$. As shown in the following figure, we know one of the incoming arcs $(a_{n_1,m}, a_{n_2,m}, a_{n_3,m})$ or the self-loop arc $a_{m,m}$ must be active at time period $t + 1$.

**Figure:** Configuration Transition Graph for Configuration $m$
Multi-Configuration Ramping Up Rate Inequalities

\[ p_{t+1}^m - \sum_{n \in C \rightarrow m} p_t^n \leq \sum_{n \in C \rightarrow m} RU^{a(n,m)} a(n,m) z_{t+1} \]

\[ - \sum_{n \in C \rightarrow m} P_n \left( \left( \sum_{a \in (A_{in}^n \cup A_{sl}^n)} z_t^a \right) - z_{t+1}^a \right), \forall m \in C, \forall t. \]  

Table: Validity of Ramping Up Inequalities (14)

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<td>( \sum_{n \in C \rightarrow m} z_{t+1}^a(n,m) )</td>
<td>( \sum_{n \in C \rightarrow m} \sum_{a \in (A_{in}^n \cup A_{sl}^n)} z_t^a )</td>
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</table>
Multi-Configuration Ramping Down Rate Inequalities

\[
\sum_{n \in C \rightarrow m} (p_t^n - p_{t+1}^m) \leq \sum_{n \in C \rightarrow m} \text{RD}^{a(n,m)} z_{t+1}^{a(n,m)} + \sum_{n \in C \rightarrow m} \overline{P}_n \left( \left( \sum_{a \in (A_{n}^{\text{in}} \cup A_{n}^{\text{sl}})} z_t^a \right) - z_{t+1}^{a(n,m)} \right), \forall \ m \in C, \forall \ t.
\] (15)

Table: Validity of Ramping Down Inequalities (15)

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<td>$\sum_{n \in C \rightarrow m} z_t^{a(n,m)}$</td>
<td>$\sum_{n \in C \rightarrow m} \sum_{a \in (A_{n}^{\text{in}} \cup A_{n}^{\text{sl}})} z_t^a$</td>
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</table>
Suppose that the combined-cycle unit works on Configuration $n$ at time period $t$ in the following figure. Then, one of the outgoing arcs $(a_{n,m_1}, a_{n,m_2}, a_{n,m_3})$ or the self-loop arc $a_{n,n}$ must be active at time period $t + 1$.

**Figure:** Configuration Transition Graph for Configuration $n$
Multi-Configuration Ramping Up Rate Inequalities

\[
\sum_{m \in C_{n \rightarrow}} p_{t+1}^m - p_t^n \leq \sum_{m \in C_{n \rightarrow}} \text{RU}^a(n, m) z_{t+1}^a(n, m) \\
+ \sum_{m \in C_{n \rightarrow}} \overline{P}_m \left( \sum_{a \in (A_{m}^{\text{in}} \cup A_{m}^{\text{sl}})} z_{t+1}^a \right) - \sum_{a \in (A_{m}^{\text{in}} \cup A_{m}^{\text{sl}})} z_{t+1}^a(n, m), \forall n \in C, \forall t.
\]

Table: Validity of Ramping Up Inequalities (16)

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</table>
Multi-Configuration Ramping Down Rate Inequalities

\[ p_n^t - \sum_{m \in C_n \rightarrow} p_{t+1}^m \leq \sum_{m \in C_n \rightarrow} \text{RD}^{a(n,m)} z_{t+1}^{a(n,m)} \]

\[ - \sum_{m \in C_n \rightarrow} P_m \left( \left( \sum_{a \in (\mathcal{A}_{in}^m \cup \mathcal{A}_{sl}^m)} z_{t+1}^a \right) - z_{t+1}^{a(n,m)} \right), \forall n \in C, \forall t. \]

Table: Validity of Ramping Down Inequalities (17)

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<td>( \sum_{m \in C_n \rightarrow} z_{t+1}^{a(n,m)} )</td>
<td>( \sum_{m \in C_n \rightarrow} \sum_{a \in (\mathcal{A}<em>{in}^m \cup \mathcal{A}</em>{sl}^m)} z_{t+1}^a )</td>
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3. Strengthened Edge-Based Formulation
   - Tighter Constraints
   - Strong Valid Inequalities

4. Case Studies

5. Conclusion
Experiment Setting

- IEEE 118 Bus System: 54 traditional thermal units and 12 combined-cycle units.
- 10 different load scenarios.
- Intel(R) Core(TM) i7-4500U 1.8GHz with 8G memory and CPLEX 12.5.
- EBF: Edge-based formulation.
- TEBF: The edge-based formulation with min-up/-down constraints (4) and (5) replaced by tighter min-up/-down constraints (8) and (9).
- REBF: The edge-based formulation with ramping constraints (6) and (7) replaced by tighter ramping constraints (10) - (17).
- SEBF: Strengthened edge-based formulation.
## Computational Results

### Table: Root Node Information

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## Computational Results

### Table: Computational Times

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Figure: Convergence evolution of Case 1 in One-Day UC
**Convergence Process**

**Figure:** Convergence evolution of Case 1 in Two-Day UC
Outline

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   - Introduction
   - Motivation

2. Edge-Based Formulation
   - Transition Graph Representation
   - Operating Constraints

3. Strengthened Edge-Based Formulation
   - Tighter Constraints
   - Strong Valid Inequalities

4. Case Studies

5. Conclusion
Contributions

- Increase the accuracy. Exactly describe the physical constraints (in particular, min-up/-down restrictions for each turbine) and transition costs between different configurations.

- Increase the flexibility by tracking the status of each turbine.

- A better computational performance.


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Thank you!