Concentric Relaxation for Transmission Switching

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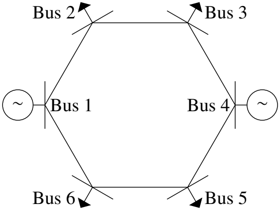
Optimal Power Flow

- Given a transmission network and a set of demands, what is the cheapest way to (a) produce the required power and (b) transmit the power to the corresponding nodes in the network.
- Power flow must obey Kirchhoff's law.
  - "Switching" lines (removing them from the network) may reduce production costs.



The Transmission Switching Problem

DC Power Flow

$$\text{Minimize} \sum_{j \in J} c_j(p_j)$$

subject to

$$\sum_{e \in E(.,i)} f_e - \sum_{e \in E(i,.)} f_e + \sum_{j \in \mathcal{N}(i)} p_j = L_i \quad \forall\, j \in V$$

$$\underline{f}_e z_e \le f_e \le \overline{f}_e z_e \quad \forall e$$

$$|f_e - B_e(\theta_i - \theta_j)| \le M_e[1 - z_e] \quad \forall e$$

$$z_e \in \{0, 1\}, \quad p_j \ge 0, \quad \theta_i, \; f_e \; \text{free.}$$

What is M?

- M_e is a large number. How large?
- The Big-M constraint enforces the logic "if line z_e is switched, then the Kirchhoff's law constraint between the adjacent buses can be ignored" (the constraint becomes redundant).
- This happens when $M_e \geq$

$$\max |f_e - B_e(\theta_i - \theta_j)|$$

$$\sum_{e' \in E(.,i)\setminus e} f_{e'} - \sum_{e' \in E'(i,.)\setminus e} f_{e'} + \sum_{j \in \mathcal{N}(i)} p_j = L_i \quad \forall\, j \in V$$

$$\underline{f}_{e'} z_{e'} \leq f_{e'} \leq \overline{f}_{e'} z_{e'} \quad \forall e' \neq e$$

$$|f_{e'} - B_{e'}(\theta_i - \theta_j)| \leq M_{e'}[1 - z_{e'}] \quad \forall e' \neq e$$

$$z_{e'} \in \{0, 1\}, \quad p_j \geq 0, \quad \theta_i, \quad f_{e'} \quad \text{free.}$$

Choosing Values of M

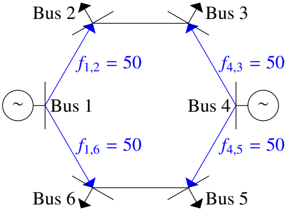
- Ideally one would want to use **the smallest** value for M_e as possible, but this requires solving an **integer programming** problem for each transmission line.
- Less ideally one would want to use **a small** value for M_e as possible, but this requires solving an **linear programming** problem for each transmission line.
- In practice, a single, large, value is used for every M_e.

Consequences of a Bad M

Big-M Constraints

$$\underline{f}_e z_e \leq f_e \leq \overline{f}_e z_e \quad \forall e \tag{1}$$

$$|f_e - B_e(\theta_i - \theta_j)| \leq M[1 - z_e] \quad \forall e \tag{2}$$



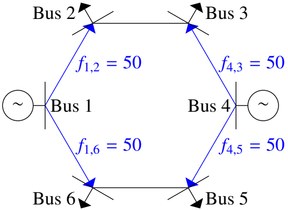
- Suppose demand at every non-generator bus is 50.
- The capacity of each line is 100.
- The susceptance of all lines is 1.

Consequences of a Bad M

Big-M Constraints

$$\underline{f}_e z_e \leq f_e \leq \overline{f}_e z_e \quad \forall e \tag{1}$$

$$|f_e - B_e(\theta_i - \theta_j)| \leq M[1 - z_e] \quad \forall e \tag{2}$$



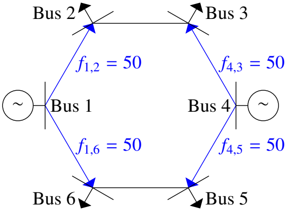
- In OPF solution, $f_{1,2} = f_{1,6} = f_{4,3} = f_{4,5} = 50$, $f_{2,3} = f_{6,5} = 0$.
- Also $z_{1,2} = z_{1,6} = z_{4,3} = z_{4,5} = 0.5$, $z_{2,3} = z_{6,5} = 0$.

Consequences of a Bad M

Big-M Constraints

$$\underline{f}_e z_e \leq f_e \leq \overline{f}_e z_e \quad \forall e \tag{1}$$

$$|f_e - B_e(\theta_i - \theta_j)| \leq M[1 - z_e] \quad \forall e \tag{2}$$



- Constraints 2 look like

$$\frac{-M}{2} \leq f_e - B_e(\theta_i - \theta_j) \leq \frac{M}{2} \tag{3}$$

for lines with flow, and

$$-M \leq f_e - B_e(\theta_i - \theta_j) \leq M \tag{4}$$

for lines without flow.

Big M is Bad M

- If M is large (which it is) these constraints are rarely going to be binding.
- Unless z_e is close to one (in the LP relaxation), power flow constraints are very weak.
- z_e is only close to one when the line is near capacity.
- It can be shown that if the transmission network contains no cycles, Kirchhoff's constraints are not necessary.
- The Kirchhoff constraints affect the LP relaxation only when several lines are fixed to be on or near capacity (enough to form a cycle).

In summary

- The current TS formulation is built on "Big-M"s. Because there are so many important Big-M constraints, the current MILP formulation of TS is not going to be solvable for any real sized network.
- Heuristics are necessary (although there is recent work looking at potentially different, Big-M-less, formulations).
- A common approach (and one taken in this work) is to "pre-screen" the list of transmission lines in order to identify which lines should be considered "switchable".

Prescreening

- Given a set, $\mathcal{S}$ of switchable lines, the relaxed TS (RTS) problem can be written as:

$$z_{\mathsf{RTS}} = \min \sum_{g \in G} c_g(p_g)$$

$$\underline{P}_g \leq p_g \leq \overline{P}_g \forall g \in \mathcal{G},$$

$$\sum_{e \in E(.,i)} f_e - \sum_{e \in E(i,.)} f_e + \sum_{j \in \mathcal{G}_i} p_j = L_i \quad \forall\ i \in V,$$

$$-\overline{f}_e \leq f_e \leq \overline{f}_e \quad \forall e \in E - \mathcal{S},$$

$$f_e = B_e(\theta_i - \theta_j) \quad \forall e = (i,j) \in E - \mathcal{S}$$

$$-\overline{f}_e z_e \leq f_e \leq \overline{f}_e z_e \quad \forall e \in \mathcal{S}$$

$$|f_e - B_e(\theta_i - \theta_j)| \leq M_e^{\mathsf{RTS}}[1 - z_e] \quad \forall e = (i,j) \in \mathcal{S},$$

Advantages of Prescreening

- Fewer binary variables! Only $|\mathcal{S}|$ switching decisions. The size of the set of switchable lines offers the user a “dial” to determine how quickly the problem is solved. Larger sets lead to more accurate solutions, but with the cost of additional computational time.
- Less obviously is that the Big-M formulation can be much tighter if several lines are not switchable. I.e., smaller values of M_e can be found for each edge. A tighter formulation should translate into faster solving times.

How to choose $\mathcal{S}$?

- We need to develop a fast way to identify which lines have the greatest potential to reduce cost when switched. To do this, we first attempt to identify (potentially) highly congested areas.
- We do this by solving the OPF problem with **no** switching as well as **no** capacity constraints on the transmission lines.
- If this gives a solution where no lines are over capacity, then the solution is feasible. Moreover, no switching is necessary!
- Assume the solution is not feasible. Let $\mathcal{V}$ be the set of transmission lines whose capacities are violated in this solution.

Concentric Relaxation and $\mathcal{S}$

- We want $\mathcal{S}$ to include transmission lines that, when switched, will divert flow away from $\mathcal{V}$. We generate these by looking at "nearby" edges (borrowing an idea by Zaborszky et al.).
- The general idea: If line e is over capacity, and e' is nearby e, then line e' is labeled switchable (is added to $\mathcal{S}$).
- How do we define "nearby"? There are lots of ways, we can use the network distance, geographic distance, etc. We define "nearby" by running many different simulations investigating the impact that switching one line has on another.

Finding Neighborhoods of a Line

Defining Nearby

Let $N_{\epsilon,\delta}(e)$ be the "neighborhood" of edge e, consisting of all edges whose flow changed by more than $\epsilon\%$ in more than $\delta\%$ of the simulations. A transmission line is in its own neighborhood.

$N_{\epsilon,\delta}(e)$ is computed by:

$$N_{\epsilon,\delta}(e) = \left\{ e' \in E \;\middle|\; \sum_{s \in S} \mathbb{1}\left\{\frac{|f^s_{e'} - f^s_{e',-e}|}{|f^s_{e'}|} \geq \frac{\epsilon}{100}\right\} \geq \frac{\delta|S|}{100} \right\},$$

Example: RTS-96

Table: $N_{0.7,0.7}$ For RTS-96

#	Adj. Buses	Lines in Relaxation								
1	(101,102)									
2	(101,103)	1								
3	(101,105)	1	2							
4	(102,104)	1	3	8	9	10	16			
5	(102,106)	1	4	8	10	16				
6	(103,109)	1	5	10						
7	(103,124)	6								
8	(104,109)	6	7	15	16	26	27			
9	(105,110)	8								
10	(106,110)	1	9	10						
11	(107,108)									
12	(107,203)	11	12	13	14	16	18	37	52	
13	(108,109)	12								
14	(108,110)									
15	(109,111)									
16	(109,112)	15								
17	(110,111)									
18	(110,112)	17	18							
19	(111,114)	18								
20	(112,113)	15	16	17	18	19	33	34		
21	(112,123)	20								
22	(113,123)	16	20	21	37					
23	(113,215)	20	22	37						
24	(114,116)	16	20	23	37					
25	(115,116)	6	15	16	17	18	19	24	33	34
26	(115,121)	6	25	26	28	30	32			
27	(115,124)									

The Algorithm

- For a given ϵ and δ, let $\mathcal{S}$ be the union of all neighbodhoods in $\mathcal{V}$. Solve the restricted TS problem where only lines in $\mathcal{S}$ are switchable.
- Pros:
  - By varying the values of ϵ and δ, you can quickly adjust the level of difficulty / precision of your problem.
  - The neighborhoods are computed in advance, so determining the set of switchable lines of very fast
- Cons:
  - The solution is not guaranteed to be optimal

Computational Results: RTS-96

- How do the values of ϵ and δ effect the speed and the quality of the TS problem?

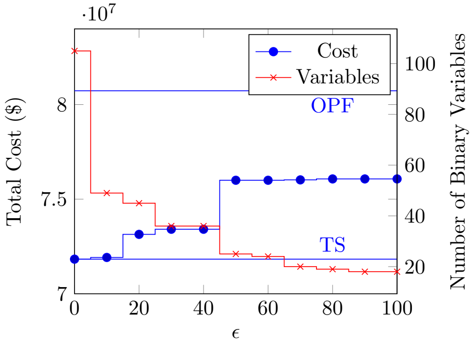


Figure: Size and Quality of $N_{\epsilon,80}$ for RTS-96

Computational Results: IEEE-662

- How do the values of ϵ and δ effect the speed and the quality of the TS problem?

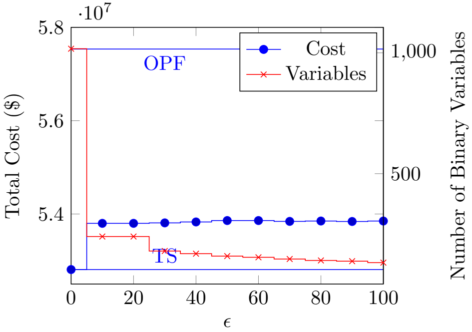


Figure: Size and Quality of $N_{\epsilon,80}$ for IEEE 662

Speedups from Prescreening

- The full RTS-96 instance is solved by CPLEX in fractions of a second, so there is no real point to performing prescreening.
- But for IEEE 662:

Table: Computational Times (s) for IEEE-662 with $\delta = 80$

ϵ	0	10	20	30	40	
Time (s)	(4.80%)	(3.02%)	(3.02%)	(0.55%)	2927	
ϵ	50	60	70	80	90	100
	2820	(0.31%)	(0.42%)	1732	411	362

Conclusion

- The TS problem is a hard integer programming problem!
- Prescreening strategies like this one can be effective at identifying lines whose switching can be most impactful.
- Prescreening can be used to reduce the size of the TS problem (in number of binary variables) by up to an order without sacrificing the objective function by *that* much.