Distributionally Robust Economic Dispatch with Dynamic Line Rating

Feng Qiu, Jianhui Wang

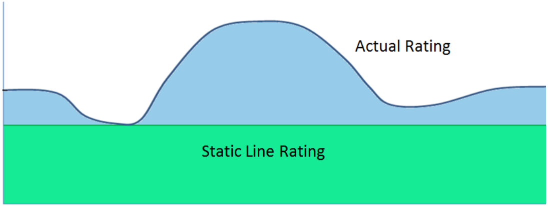
Decision & Information Science Division
Argonne National Laboratory

FERC Software Conference
June 25, 2014

Outline

Thermal Limits of Overhead Transmission Lines

1. Line rating: maximal allowable currents on a transmission line
2. Actual rating heavily depends on ambient temperature, solar radiation, and wind speed.
   - Example (The Valley Group 11'): 20 mile transmission line (795 ACSR)
   - Ambient temperature $\downarrow$ 10 °C $\Rightarrow$ $\uparrow$ 11% capacity
   - Wind speed (90°) $\uparrow$ 1m/sec $\Rightarrow$ $\uparrow$ 44% capacity
3. <u>*Static line rating*</u>
   - Protect against annealing, reliability, and security risk; manufacturing error
   - However, very conservative



Dynamic Line Rating

- *Dynamic Line Rating (DLR):*
  - Monitor real-time ambient environment (e.g., temperature, wind speed, line tension)
  - Forecast real-time transmission capacity
- Benefits by DLR
  - Reduce operator intervention and increase grid reliability
  - Help wind integration and reduce curtailment
  - Relieve Contingency, improve economical dispatch
  - ......
- Focus of this study
  - Study overloading risks caused by DLR forecast errors
  - Incorporate overloading risk control in DLR applications
- Inspired by the Valley Group presentation at FERC technical conference in 2013

Dynamic Line Rating

- *Dynamic Line Rating (DLR):*
  - Monitor real-time ambient environment (e.g., temperature, wind speed, line tension)
  - Forecast real-time transmission capacity
- Benefits by DLR
  - Reduce operator intervention and increase grid reliability
  - Help wind integration and reduce curtailment
  - Relieve Contingency, improve economical dispatch
  - ......
- Focus of this study
  - Study overloading risks caused by DLR forecast errors
  - Incorporate overloading risk control in DLR applications
- Inspired by the Valley Group presentation at FERC technical conference in 2013

Dynamic Line Rating

- *Dynamic Line Rating (DLR):*
  - Monitor real-time ambient environment (e.g., temperature, wind speed, line tension)
  - Forecast real-time transmission capacity
- Benefits by DLR
  - Reduce operator intervention and increase grid reliability
  - Help wind integration and reduce curtailment
  - Relieve Contingency, improve economical dispatch
  - ......
- Focus of this study
  - Study overloading risks caused by DLR forecast errors
  - Incorporate overloading risk control in DLR applications
- Inspired by the Valley Group presentation at FERC technical conference in 2013

Dynamic Line Rating

- *Dynamic Line Rating (DLR):*
  - Monitor real-time ambient environment (e.g., temperature, wind speed, line tension)
  - Forecast real-time transmission capacity
- Benefits by DLR
  - Reduce operator intervention and increase grid reliability
  - Help wind integration and reduce curtailment
  - Relieve Contingency, improve economical dispatch
  - ......
- Focus of this study
  - Study overloading risks caused by DLR forecast errors
  - Incorporate overloading risk control in DLR applications
- Inspired by the Valley Group presentation at FERC technical conference in 2013

An Application of DLR on Economic Dispatch

- Economic dispatch with DLR options
- Forecasted line rating: α_ℓ (percentage) extra capacity on line ℓ
- Decision x_ℓ: whether to use the extra capacity α_ℓ on line ℓ

ED with DLR Formulation

$$\min \sum_{g \in G} \sum_{t \in T} c_g(p_{g,t}) + \sum_{n \in N} \sum_{t \in T} h_{n,t} q_{n,t}$$

$$\text{s.t. } G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0$$

$$-SLR_\ell \cdot (1 + \alpha_{\ell,t} x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1 + \alpha_{\ell,t} x_{\ell,t}) \; \forall \ell \in L \; \forall t \in T$$

- $p_{g,t}$ power generation; $h_{n,t}$ load shedding at bus n
- G: all constraints in a regular economic dispatch formulation
- SLR: static line rating

Forecast Errors

- Forecasting:
  - A value/interval with a probability: at least α_ℓ extra capacity with a probability p_ℓ
  - Forecast errors are inherent
- Consequences of forecast errors
  - Security issues
  - Cost incurred by redispatching
- Overloading risk on a single line
  - Kim & Dobson 2011, Zhang, Pu, et al. 2002, Wan, McCalley, and Vittal 1999, etc.
- Overloading risk on multiple lines
  - Forecast errors are correlated, e.g., local weather changes
  - Redispatching/rerouting power becomes significantly more difficult
  - Current N-1 contingency does not capture multiple-line trips

Forecast Errors

- Forecasting:
  - A value/interval with a probability: at least α_ℓ extra capacity with a probability p_ℓ
  - Forecast errors are inherent
- Consequences of forecast errors
  - Security issues
  - Cost incurred by redispatching
- Overloading risk on a single line
  - Kim & Dobson 2011, Zhang, Pu, et al. 2002, Wan, McCalley, and Vittal 1999, etc.
- Overloading risk on multiple lines
  - Forecast errors are correlated, e.g., local weather changes
  - Redispatching/rerouting power becomes significantly more difficult
  - Current N-1 contingency does not capture multiple-line trips

Forecast Errors

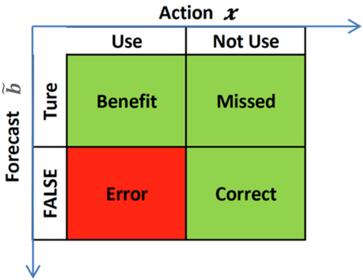
- Forecasting:
  - A value/interval with a probability: at least α_ℓ extra capacity with a probability p_ℓ
  - Forecast errors are inherent
- Consequences of forecast errors
  - Security issues
  - Cost incurred by redispatching
- Overloading risk on a single line
  - Kim & Dobson 2011, Zhang, Pu, et al. 2002, Wan, McCalley, and Vittal 1999, etc.
- Overloading risk on multiple lines
  - Forecast errors are correlated, e.g., local weather changes
  - Redispatching/rerouting power becomes significantly more difficult
  - Current N-1 contingency does not capture multiple-line trips

Forecast Errors

- Forecasting:
  - A value/interval with a probability: at least α_ℓ extra capacity with a probability p_ℓ
  - Forecast errors are inherent
- Consequences of forecast errors
  - Security issues
  - Cost incurred by redispatching
- Overloading risk on a single line
  - Kim & Dobson 2011, Zhang, Pu, et al. 2002, Wan, McCalley, and Vittal 1999, etc.
- Overloading risk on multiple lines
  - Forecast errors are correlated, e.g., local weather changes
  - Redispatching/rerouting power becomes significantly more difficult
  - Current N-1 contingency does not capture multiple-line trips

Forecast Errors

- Model forecast errors
  - $\tilde{b}_\ell$: Bernoulli random number, whether line ℓ has α extra capacity
  - $\tilde{b}_\ell = 1$: forecast is correct
- Outcome table



  - when $\tilde{b}_\ell = 0; x_\ell = 1$, potential overloading risk

A Risk Measure for Overloading on Multiple Lines

Definition

The probability that more than k lines are at an overloading risk

- k: parameter chosen based on system configuration, operator experience, etc.

Risk Requirement

$$\mathbb{P}\,(\text{more than } k \text{ lines are at an overloading risk}) \leq \epsilon.$$

$$\Downarrow$$

$$\mathbb{P}\left(\sum_{\ell \in L}(1-\tilde{b}_{\ell,t})x_{\ell,t} \geq k+1\right) \leq \epsilon.$$

- $\epsilon \in (0,1)$: operator's tolerance on the risk level
- Can be very general for modeling decisions with forecast errors

A Risk Measure for Overloading on Multiple Lines

Definition

The probability that more than k lines are at an overloading risk

- k: parameter chosen based on system configuration, operator experience, etc.

Risk Requirement

$$\mathbb{P}\left(\text{more than } k \text{ lines are at an overloading risk}\right) \leq \epsilon.$$

$$\Downarrow$$

$$\mathbb{P}\left(\sum_{\ell \in L} (1 - \tilde{b}_{\ell,t}) x_{\ell,t} \geq k + 1\right) \leq \epsilon.$$

- $\epsilon \in (0, 1)$: operator's tolerance on the risk level
- Can be very general for modeling decisions with forecast errors

A Risk Measure for Overloading on Multiple Lines

Definition

The probability that more than k lines are at an overloading risk

- k: parameter chosen based on system configuration, operator experience, etc.

Risk Requirement

$$\mathbb{P}\,(\text{more than } k \text{ lines are at an overloading risk}) \leq \epsilon.$$

$$\Downarrow$$

$$\mathbb{P}\left(\sum_{\ell \in L}(1-\tilde{b}_{\ell,t})x_{\ell,t} \geq k+1\right) \leq \epsilon.$$

- $\epsilon \in (0,1)$: operator's tolerance on the risk level
- Can be very general for modeling decisions with forecast errors

A Risk Measure for Overloading on Multiple Lines

Definition

The probability that more than k lines are at an overloading risk

- k: parameter chosen based on system configuration, operator experience, etc.

Risk Requirement

$$\mathbb{P}\left(\text{more than } k \text{ lines are at an overloading risk}\right) \leq \epsilon.$$

$$\Downarrow$$

$$\mathbb{P}\left(\sum_{\ell \in L} (1 - \tilde{b}_{\ell,t}) x_{\ell,t} \geq k + 1\right) \leq \epsilon.$$

- $\epsilon \in (0, 1)$: operator's tolerance on the risk level
- Can be very general for modeling decisions with forecast errors

Ambiguity When Information is Incomplete

- Challenges in evaluating the risk:
  - Incomplete information. Only joint distributions up to level m are known, e.g., marginal and pair-wise joint distributions ($m = 2$)
  - Complete distribution data is of exponential size
- Ambiguity occurs when information is incomplete
  - Distribution function CANNOT be uniquely determined
  - A single distribution ξ V.S. a family of distributions $\mathcal{P}$
  - Which one to use to evaluate the probability? It is ambiguous
- To clarify the ambiguity: a *worst-case* point of view

Distributionally Robust Model

$$X := \{x \in \{0,1\}^L : \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_\xi \left(\sum_{\ell \in L} (1 - \tilde{b}_\ell) x_\ell \geq k + 1 \right) \right) \leq \epsilon \}$$

Ambiguity When Information is Incomplete

- Challenges in evaluating the risk:
  - Incomplete information. Only joint distributions up to level m are known, e.g., marginal and pair-wise joint distributions ($m = 2$)
  - Complete distribution data is of exponential size
- Ambiguity occurs when information is incomplete
  - Distribution function CANNOT be uniquely determined
  - A single distribution ξ V.S. a family of distributions $\mathcal{P}$
  - Which one to use to evaluate the probability? It is ambiguous
- To clarify the ambiguity: a *worst-case* point of view

Distributionally Robust Model

$$X := \{x \in \{0,1\}^L : \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_{\xi}(\sum_{\ell \in L} (1 - \tilde{b}_\ell) x_\ell \geq k+1) \right) \leq \epsilon\}$$

Ambiguity When Information is Incomplete

- Challenges in evaluating the risk:
  - Incomplete information. Only joint distributions up to level m are known, e.g., marginal and pair-wise joint distributions ($m = 2$)
  - Complete distribution data is of exponential size
- Ambiguity occurs when information is incomplete
  - Distribution function CANNOT be uniquely determined
  - A single distribution ξ V.S. a family of distributions $\mathcal{P}$
  - Which one to use to evaluate the probability? It is ambiguous
- To clarify the ambiguity: a *worst-case* point of view

Distributionally Robust Model

$$X := \{x \in \{0,1\}^L : \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_{\xi} \left(\sum_{\ell \in L} (1 - \tilde{b}_{\ell}) x_{\ell} \geq k + 1 \right) \right) \leq \epsilon \}$$

Economic Dispatch with DLR

- Dispatch in the look-ahead model with DLR
  - Multi-period economic dispatch
  - Dynamic line rating forecasts for line capacities
- Use only those DLR forecast such that
  - Generation and load shedding costs are reduced most effectively
  - Overloading risk requirement is satisfied
- The mathematical model

$$\min \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t}q_{n,t}$$

$$\text{s.t. } G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0$$

$$-SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \quad \forall \ell \in L\, \forall t \in T$$

$$\left[\sup_{\xi\in\mathcal{P}} \left(\mathbb{P}_\xi\big(\sum_{\ell\in L}(1-\tilde{b}_{\ell,t})x_{\ell,t} \geq k+1\big)\right) \leq \epsilon\right] \quad \forall t \in T$$

Economic Dispatch with DLR

- Dispatch in the look-ahead model with DLR
  - Multi-period economic dispatch
  - Dynamic line rating forecasts for line capacities
- Use only those DLR forecast such that
  - Generation and load shedding costs are reduced most effectively
  - Overloading risk requirement is satisfied
- The mathematical model

$$\min \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t}q_{n,t}$$

$$\text{s.t. } G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0$$

$$-SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \quad \forall \ell \in L\, \forall t \in T$$

$$\left[\sup_{\xi\in\mathcal{P}} \left(\mathbb{P}_\xi(\sum_{\ell\in L}(1-\tilde{b}_{\ell,t})x_{\ell,t} \geq k+1)\right) \leq \epsilon\right] \quad \forall t \in T$$

Economic Dispatch with DLR

- Dispatch in the look-ahead model with DLR
  - Multi-period economic dispatch
  - Dynamic line rating forecasts for line capacities
- Use only those DLR forecast such that
  - Generation and load shedding costs are reduced most effectively
  - Overloading risk requirement is satisfied
- The mathematical model

$$
\begin{aligned}
\min & \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t} q_{n,t} \\
\text{s.t. } & G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0 \\
& -SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \quad \forall \ell \in L\; \forall t \in T \\
& \left[\sup_{\xi\in\mathcal{P}} \left(\mathbb{P}_\xi(\sum_{\ell\in L}(1-\tilde{b}_{\ell,t})x_{\ell,t} \geq k+1)\right) \leq \epsilon\right] \quad \forall t \in T
\end{aligned}
$$

An Inner Approximation

- Identifying the worst case distribution requires exponential-size data
  - The Boolean problem: exponential size
- Construction of an inner approximation of X
  - Let $U(x) \geq F(x) := \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_{\xi}(\sum_{\ell \in L}(1 - \tilde{b}_\ell)x_\ell \geq k+1) \right)$ for any x of interest

Inner Approximation

Let $\bar{X} := \{x \in \{0,1\}^L : U(x) \leq \epsilon\}$, then

$$\bar{X} \subseteq X$$

- $U(x)$ has to be *computable*
- $U(x)$ should be as *tight* as possible

An Inner Approximation

- Identifying the worst case distribution requires exponential-size data
  - The Boolean problem: exponential size
- Construction of an inner approximation of X
  - Let $U(x) \geq F(x) := \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_{\xi}(\sum_{\ell \in L}(1 - \tilde{b}_\ell)x_\ell \geq k + 1) \right)$ for any x of interest

Inner Approximation

Let $\bar{X} := \{x \in \{0,1\}^L : U(x) \leq \epsilon\}$, then

$$\bar{X} \subseteq X$$

- $U(x)$ has to be *computable*
- $U(x)$ should be as *tight* as possible

An Inner Approximation

- Identifying the worst case distribution requires exponential-size data
  - The Boolean problem: exponential size
- Construction of an inner approximation of X
  - Let $U(x) \geq F(x) := \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_{\xi} (\sum_{\ell \in L} (1 - \tilde{b}_\ell) x_\ell \geq k + 1) \right)$ for any x of interest

Inner Approximation

Let $\bar{X} := \{x \in \{0,1\}^L : U(x) \leq \epsilon\}$, then

$$\bar{X} \subseteq X$$

  - $U(x)$ has to be *computable*
  - $U(x)$ should be as *tight* as possible

An Inner Approximation

- Identifying the worst case distribution requires exponential-size data
  - The Boolean problem: exponential size
- Construction of an inner approximation of X
  - Let $U(x) \geq F(x) := \sup_{\xi \in \mathcal{P}} \left(\mathbb{P}_{\xi} (\sum_{\ell \in L} (1 - \tilde{b}_{\ell}) x_{\ell} \geq k + 1) \right)$ for any x of interest

Inner Approximation

Let $\bar{X} := \{x \in \{0, 1\}^L : U(x) \leq \epsilon\}$, then

$$\bar{X} \subseteq X$$

  - $U(x)$ has to be *computable*
  - $U(x)$ should be as *tight* as possible

Linear Programming (LP)-Based Probability Bounds

- $(1-\tilde{b}_\ell)x_\ell$ can be treated as a Bernoulli random number parameterized by x_ℓ
- $F(x)$: *the probability that at least $k+1$ events occur*
  - $\mathbb{P}(\ (1-\tilde{b}_\ell)x_\ell\ =\ 1\ ) = (1-p_\ell)x_\ell$
  - $\mathbb{P}(\ (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1, (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1\ ) = \bar{p}_{\ell_1,\ell_2}x_{\ell_1}x_{\ell_2}$

Proposition (Prékopa, 1990. A LP-Based Probability Bound)

$$U(x) = \max\{\sum_{j=k+1}^{|L|} v_j : \sum_{j=i}^{|L|} \binom{j}{i} v_j = s_i(x)\ i = 0 \ldots m\}$$

- $s_0(x) = 1$, $s_i(x) = \sum_{C\subseteq L:|C|=i} p_C \prod_{j\in C} x_j$, and $\binom{j}{0} = 1$; $v_i \geq 0$
- A disaggregated LP provides better bounds [Prékopa & Gao, 2005]
- The bounds above can be significantly improved by adding a set of linear inequalities [Qiu, Ahmed, & Dey, 2013]

Linear Programming (LP)-Based Probability Bounds

- $(1-\tilde{b}_\ell)x_\ell$ can be treated as a Bernoulli random number parameterized by x_ℓ
- $F(x)$: *the probability that at least $k+1$ events occur*
  - $\mathbb{P}(\ (1-\tilde{b}_\ell)x_\ell\ =\ 1\ ) = (1-p_\ell)x_\ell$
  - $\mathbb{P}(\ (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1, (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1\ ) = \bar{p}_{\ell_1,\ell_2}x_{\ell_1}x_{\ell_2}$

Proposition (Prékopa, 1990. A LP-Based Probability Bound)

$$U(x) = \max\{\sum_{j=k+1}^{|L|} v_j : \sum_{j=i}^{|L|} \binom{j}{i} v_j = s_i(x)\ i = 0...m\}$$

- $s_0(x) = 1$, $s_i(x) = \sum_{C\subseteq L:|C|=i} p_C \prod_{j\in C} x_j$, and $\binom{j}{0} = 1$; $v_i \geq 0$

- A disaggregated LP provides better bounds [Prékopa & Gao, 2005]
- The bounds above can be significantly improved by adding a set of linear inequalities [Qiu, Ahmed, & Dey, 2013]

Linear Programming (LP)-Based Probability Bounds

- $(1-\tilde{b}_\ell)x_\ell$ can be treated as a Bernoulli random number parameterized by x_ℓ
- $F(x)$: *the probability that at least $k+1$ events occur*
  - $\mathbb{P}(\ (1-\tilde{b}_\ell)x_\ell\ =\ 1\ ) = (1-p_\ell)x_\ell$
  - $\mathbb{P}(\ (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1, (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1\ ) = \bar{p}_{\ell_1,\ell_2}x_{\ell_1}x_{\ell_2}$

Proposition (Prékopa, 1990. A LP-Based Probability Bound)

$$U(x) = \max\{\sum_{j=k+1}^{|L|} v_j : \sum_{j=i}^{|L|} \binom{j}{i} v_j = s_i(x)\ i = 0...m\}$$

- $s_0(x) = 1$, $s_i(x) = \sum_{C\subseteq L:|C|=i} p_C \prod_{j\in C} x_j$, and $\binom{j}{0} = 1$; $v_i \geq 0$

- A disaggregated LP provides better bounds [Prékopa & Gao, 2005]
- The bounds above can be significantly improved by adding a set of linear inequalities [Qiu, Ahmed, & Dey, 2013]

Linear Programming (LP)-Based Probability Bounds

- $(1-\tilde{b}_\ell)x_\ell$ can be treated as a Bernoulli random number parameterized by x_ℓ
- $F(x)$: *the probability that at least $k+1$ events occur*
  - $\mathbb{P}(\ (1-\tilde{b}_\ell)x_\ell\ =\ 1\ ) = (1-p_\ell)x_\ell$
  - $\mathbb{P}(\ (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1, (1-\tilde{b}_{\ell_1})x_{\ell_1}\ =\ 1\ ) = \bar{p}_{\ell_1,\ell_2}x_{\ell_1}x_{\ell_2}$

Proposition (Prékopa, 1990. A LP-Based Probability Bound)

$$U(x) = \max\{\sum_{j=k+1}^{|L|} v_j : \sum_{j=i}^{|L|} \binom{j}{i} v_j = s_i(x)\ i = 0...m\}$$

- $s_0(x) = 1$, $s_i(x) = \sum_{C\subseteq L:|C|=i} p_C \prod_{j\in C} x_j$, and $\binom{j}{0} = 1$; $v_i \geq 0$
- A disaggregated LP provides better bounds [Prékopa & Gao, 2005]
- The bounds above can be significantly improved by adding a set of linear inequalities [Qiu, Ahmed, & Dey, 2013]

Linearization

- Step 1:

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \max\{e^\top v : T^\top v = S(x)\} \leq \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \min\{\pi^\top S(x) : \pi^\top T \geq e_k^\top\} \leq \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \exists \pi \in \mathbb{R}^{m+1} : \pi^\top S(x) \leq \epsilon, \pi^\top T \geq e_k^\top\}$$

- Step 2:
  - Nonlinear terms $y_C := \pi_i \prod_{j \in C} x_j$
  - McCormick linearization technique:

$$\begin{aligned} y_C &\leq M^+ x_j \quad \forall j \in C, \\ y_C &\geq -M^- x_j \quad \forall j \in C, \\ y_C &\leq \pi_t + M^+(|C| - \textstyle\sum_{j \in C} x_j) \\ y_C &\geq \pi_t - M^-(|C| - \textstyle\sum_{j \in C} x_j), \end{aligned}$$

  - M can be properly bounded [Qiu, 2013]

Linearization

- Step 1:

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \max\{e^\top v : T^\top v = S(x)\} \leq \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \min\{\pi^\top S(x) : \pi^\top T \geq e_k^\top\} \leq \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \exists \pi \in \mathbb{R}^{m+1} : \pi^\top S(x) \leq \epsilon, \pi^\top T \geq e_k^\top\}$$

- Step 2:
  - Nonlinear terms $y_C := \pi_i \prod_{j \in C} x_j$
  - McCormick linearization technique:

$$\begin{aligned} y_C &\leq M^+ x_j \quad \forall j \in C, \\ y_C &\geq -M^- x_j \quad \forall j \in C, \\ y_C &\leq \pi_t + M^+(|C| - \textstyle\sum_{j \in C} x_j) \\ y_C &\geq \pi_t - M^-(|C| - \textstyle\sum_{j \in C} x_j), \end{aligned}$$

  - M can be properly bounded [Qiu, 2013]

Linearization

- Step 1:

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \max\{e^\top v : T^\top v = S(x)\} \le \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \min\{\pi^\top S(x) : \pi^\top T \ge e_k^\top\} \le \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \exists \pi \in \mathbb{R}^{m+1} : \pi^\top S(x) \le \epsilon, \pi^\top T \ge e_k^\top\}$$

- Step 2:
  - Nonlinear terms $y_C := \pi_i \prod_{j \in C} x_j$
  - McCormick linearization technique:

$$\begin{aligned} y_C &\le M^+ x_j \quad \forall j \in C, \\ y_C &\ge -M^- x_j \quad \forall j \in C, \\ y_C &\le \pi_t + M^+(|C| - \textstyle\sum_{j \in C} x_j) \\ y_C &\ge \pi_t - M^-(|C| - \textstyle\sum_{j \in C} x_j), \end{aligned}$$

  - M can be properly bounded [Qiu, 2013]

Linearization

- Step 1:

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \max\{e^\top v : T^\top v = S(x)\} \leq \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \min\{\pi^\top S(x) : \pi^\top T \geq e_k^\top\} \leq \epsilon\}$$

$$\Downarrow$$

$$\bar{X} = \{x \in \{0,1\}^{|L|} : \exists \pi \in \mathbb{R}^{m+1} : \pi^\top S(x) \leq \epsilon, \pi^\top T \geq e_k^\top\}$$

- Step 2:
  - Nonlinear terms $y_C := \pi_i \prod_{j \in C} x_j$
  - McCormick linearization technique:

$$\begin{aligned} y_C &\leq M^+ x_j \quad \forall j \in C, \\ y_C &\geq -M^- x_j \quad \forall j \in C, \\ y_C &\leq \pi_t + M^+(|C| - \textstyle\sum_{j \in C} x_j) \\ y_C &\geq \pi_t - M^-(|C| - \textstyle\sum_{j \in C} x_j), \end{aligned}$$

  - M can be properly bounded [Qiu, 2013]

Mixed-Integer Linear Program Formulation

- MILP Formulation

$$
\begin{aligned}
\min & \sum_{g\in G}\sum_{t\in T} c_g(p_{g,t}) + \sum_{n\in N}\sum_{t\in T} h_{n,t}q_{n,t} \\
\text{s.t. } & G(p_{g,t}, p_{\ell,t}, q_{n,t}) \geq 0 \\
& -SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \leq p_{\ell,t} \leq SLR_\ell \cdot (1+\alpha_{\ell,t}x_{\ell,t}) \ \forall \ell \in L \ \forall t \in T \\
& \pi_0 + \sum_{C\subseteq L:|C|\leq m} p_C y_C \leq \epsilon \\
& \pi_0 + \sum_{t=i}^{m} \binom{i}{t} \pi_t \leq e_k^i \quad i = 1, \ldots n \\
& -M^- x_j \leq y_C \leq M^+ x_j \quad \forall j \in C, \forall C \subseteq L : |C| \leq m \\
& \pi_t - M^-(|C| - \sum_{j\in C} x_j) \leq y_C \geq -M^- x_j \quad \forall j \in C, \forall C \subseteq L : |C| \leq m
\end{aligned}
$$

A Case Study – Dispatch Cost Reduction

- Experiment settings
  - IEEE 73 (RTS 96)-bus system
  - High loads, insufficient generation
  - 4-time-period economic dispatch
  - $m = 2$, i.e., only marginal and pair-wise joint distributions are available
  - $k = 3$, evaluating the overloading risk on 3 or more lines
  - Two sets of rating forecast data:
    - lower ratings (15% over static rating) with higher confidence levels
    - higher ratings (30% over static rating) but with lower confidence levels

Case Study – Dispatch Cost Reduction

Table: Comparison of Load Shedding Reduction

Threshold (α)	ϵ	L.S. Reduction	Avg. # of Lines Used	k-Overloading Risk*
0.15	0.01	67%	5.25	0.007
	0.05	69%	6.25	0.016
0.30	0.01	68%	3.25	0.009
	0.05	80%	5.00	0.030

k-Overloading Risk: an upper bound on the actual overloading risk

- Observations:
  - Comparing with no risk control (α=0.15): L.S. Reduction = 100%, Avg. # of lines=12, but 3-overloading risk = 0.08
  - Overloading risk under control; load shedding cost reduced
  - More risks, more gains
  - For the same risk level requirement, the lower rating data set has a larger set of lines to utilize the extra capacity predicted by DLR than the higher rating data set
  - Similar patterns observed in thermal generation cost reduction during normal operation conditions.

Conclusions and Future Research

- Conclusions
  - Risk measure for overloading risk on multiple lines caused by DLR forecast errors
  - Distributionally robust economic dispatch model with DLR
  - Mixed-integer program formulation
- Future research
  - Develop more compact MILP formulations; efficient algorithms
  - Other perspectives on overloading risk

Thank you!

Comments?