On the role of wind covariance estimation in power grid dispatch

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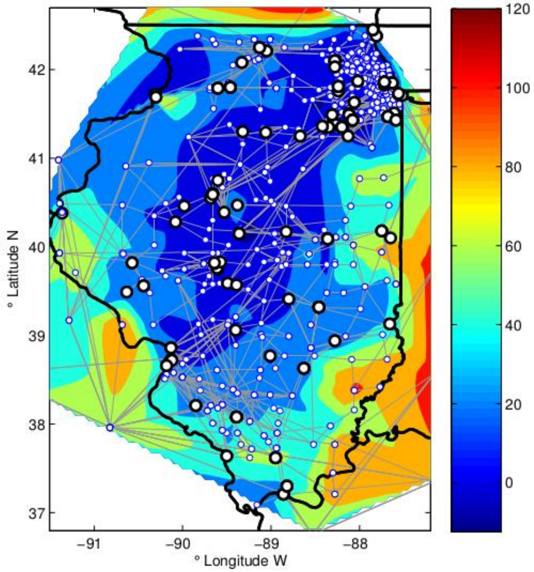
Outline

- Economic dispatch models in power grid
- Stochastic optimization models
- Wind volatility, weather forecasting and (re)sampling
- Covariance estimation and impact on optimal dispatch
  - Simple model
  - Simulation of economic dispatch for Illinois power grid
- Conclusions

Economic dispatch models

- Basis for the electricity distribution and electricity market
- Used by all Independent System Operators (ISOs) in the US.
- In the simpler form, for direct currents, is formulated as a linear programming problem

$$
\begin{aligned}
\min_{x,f} \quad & \sum_{i \in G} c_i x_i \\
\text{subj.to:} \quad & \tau_n(f) + \sum_{i \in T(n)} x_i = d_n, \forall n \in N \\
& f \in U \\
& x_i \in C_i, \forall i \in G
\end{aligned}
$$



- For alternating currents (AC), it takes the form of power flow, a nonlinear programming problem

Stochastic dispatch models

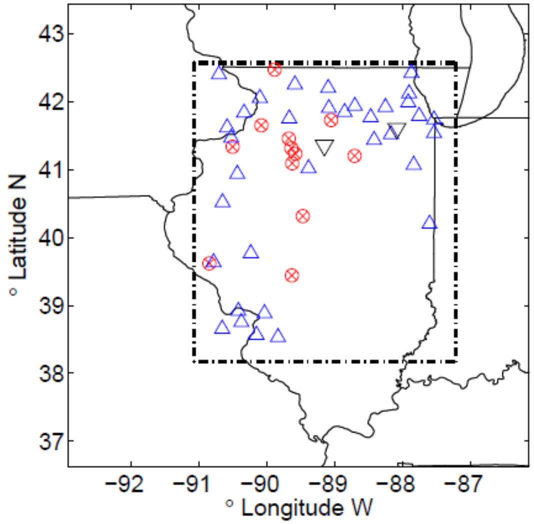
- Adoption of highly volatile renewable energy and randomness in demand requires stochastic formulations
- Cost-optimal decision in the presence of uncertain generation/demand
- Two-stage linear stochastic programming with recourse: "energy only" model (Pritchard, Zakeri, Philpott, 2010)

$$
\begin{aligned}
\min_{x, X(\omega), f, F(\omega)} \quad & \sum_{i \in G} c_i x_i + \mathbb{E}_\omega \sum_{i \in G} p_i |x_i - y_i(\omega)| \\
\text{subj.to:} \quad & \tau_n(f) + \sum_{i \in T(n)} x_i = d_n, \forall n \in N \\
& \tau_n(F(\omega)) + \sum_{i \in T(n)} y_i(\omega) = d_n, \forall n \in N, \omega \in \Omega \\
& f, F(\omega) \in U, \forall \omega \in \Omega \\
& x_i, y_i(\omega) \in U_i, \forall i \in G, \omega \in \Omega
\end{aligned}
$$

- Two-markets: "ahead", decisions/prices to be taken-now and "realtime", scenario-specific adjustments in decisions/prices.
- The model is ISO revenue adequate (no "missing money").

Integrating wind samples in the economic dispatch model

- The probability distributions are usually not known, and sampling is used (Ω is finite in practice).
- Numerical weather forecasting is needed to obtain wind samples.
- **Approach 1**: Wind farms bid energy based on their own, independent forecasts. The ISO then considers all the scenarios in the ED model.
  - Correlation among wind farms is lost
  - An exhaustive list of scenarios leads to a gigantic ED problem. Not clear how to bundle scenarios to reduce dimensionality.
- **Approach 2**: Centralized forecast at the ISO level



- Here we show that Approach 2 should be considered: ignoring or missing correlation information leads to inefficient dispatch.

Motivating example – role of correlation in dispatch

- A very simplistic model: 3 generators (of which 2 wind farms and 1 thermal), 1 demand node, no line constraints
- Power outputs of the wind farms are $W_1 \sim \mathcal{N}(w_1, \sigma_1)$ and $W_2 \sim \mathcal{N}(w_2, \sigma_2)$, and the correlation is ρ ($\rho = \mathbb{E}[(W_1 - w_1)(W_2 - w_2)]/(\sigma_1\sigma_2)$).

- **How does correlation affect the optimal dispatch cost?**
- The optimization problem can be solved analytically, and the (expected) optimal dispatch cost is:

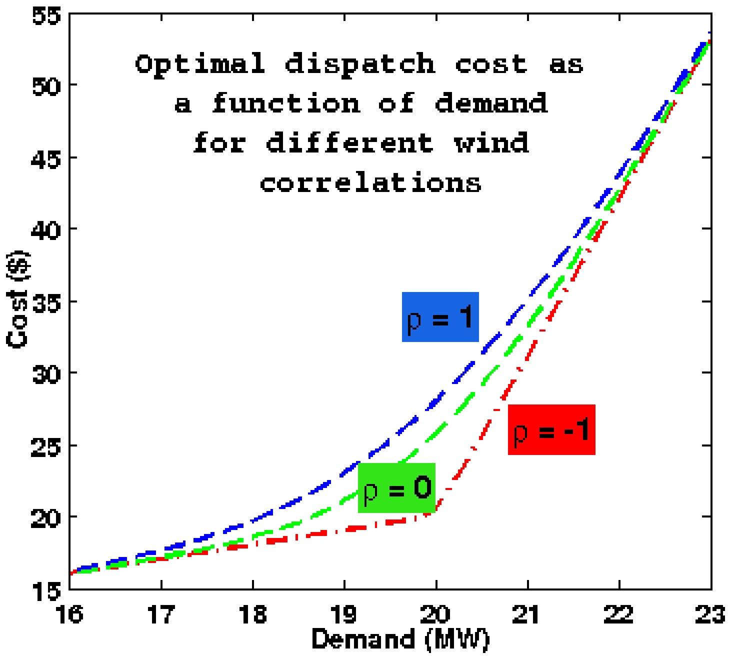
$$c_d(\rho) = c_w d + (c_{th} - c_w)((d - w_1 - w_2)\Phi(d, \sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2) + \sigma^2\phi(d, \sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2))$$

- Here Φ and ϕ are the cumulative distribution and probability distribution functions of $W = W_1 + W_2$

- The optimal dispatch cost is an **increasing function** of the correlation ρ !

Motivating example - continued

- Not accounting for positive correlation leads to an “optimistic dispatch” (more wind power is thought to be available).
- Ignoring negative correlations results in an “pessimistic dispatch” (foreseen wind is low, therefore, more thermal generation is dispatched).



- In both cases higher operating costs are obtained over time:
  - “optimistic” case potentially ends in using expensive power from the reserves to replace the wind that was predicted but not realized.
  - “pessimistic” case has higher dispatch cost since more thermal generation than necessary is dispatched.

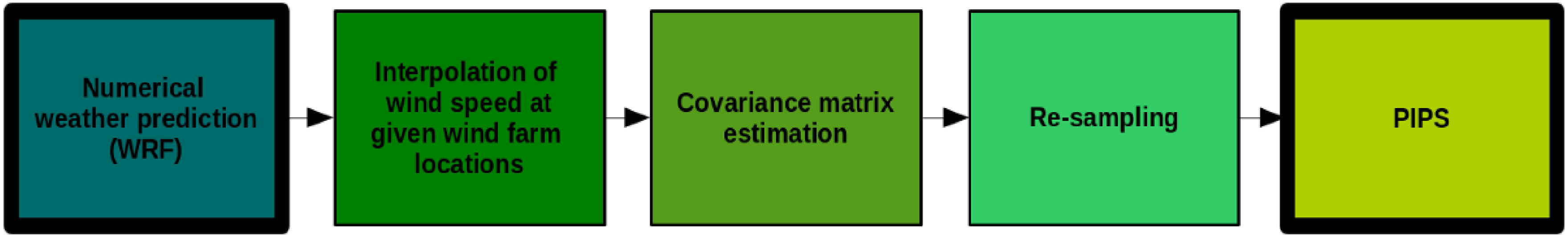
- Also leads to arbitrage opportunities in the power market for participants that account or have better approximation of the correlation.

A framework for stochastic economic dispatch

- **What about real-world large-scale power grid systems?**

- Analytical analysis of such complex systems is virtually impossible.
- Computer simulations are needed.

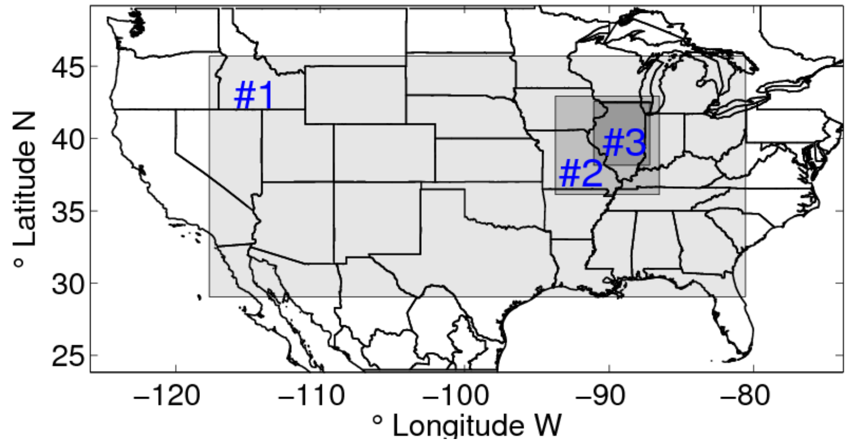
- Weather forecasting is integrated with decision making under the same computational framework.



- Wind samples using WRF, resampling using shrinkage estimators (more later).
- PIPS (Petra et all) - parallel optimization solver for high performance computing platforms (BG/P, BG/Q, Cray XE6, XC30, XK7).

Wind forecast

- Weather forecasting @ Argonne (E. Constantinescu)



- WRF (Weather Research and Forecasting) Model
  - Real-time grid-nested simulation using atmospheric models
  - Done on high performance computing platforms but still computationally expensive
  - Only 30 samples or less can be obtained in times compatible with operational practice

Covariance estimation

- A small number of samples may not accurately capture the uncertainty.
- We assume Gaussian distribution of wind speeds and resample to generate more samples.
- The statistical problem: estimate the covariance matrix Q a random p-dimensional vector based on a number of n samples
  - $X = [x_1; x_2; \cdots x_n] \in \mathbb{R}^{p \times n}$ are the samples
  - Let $\bar{x} = (x_1 + x_2 + \ldots + x_n)/n$ denote the sample mean
  - An estimator of the covariance matrix would then be

$$S = \frac{1}{n}\left[\sum_{i=1}^{n} (\mathbf{x}_i - \overline{\mathbf{x}}) \cdot (\mathbf{x}_i - \overline{\mathbf{x}})^{\mathrm{t}}\right] \in \mathbb{R}^{p \times p}$$

- Estimating covariance matrix is an issue in this situation since the number of samples (n=30) is smaller than the number of random variables (p=O(100)).

Shrinkage estimators

- Estimators of the form

$$S_\Theta = \rho_1 \cdot I + \rho_2 \cdot S$$

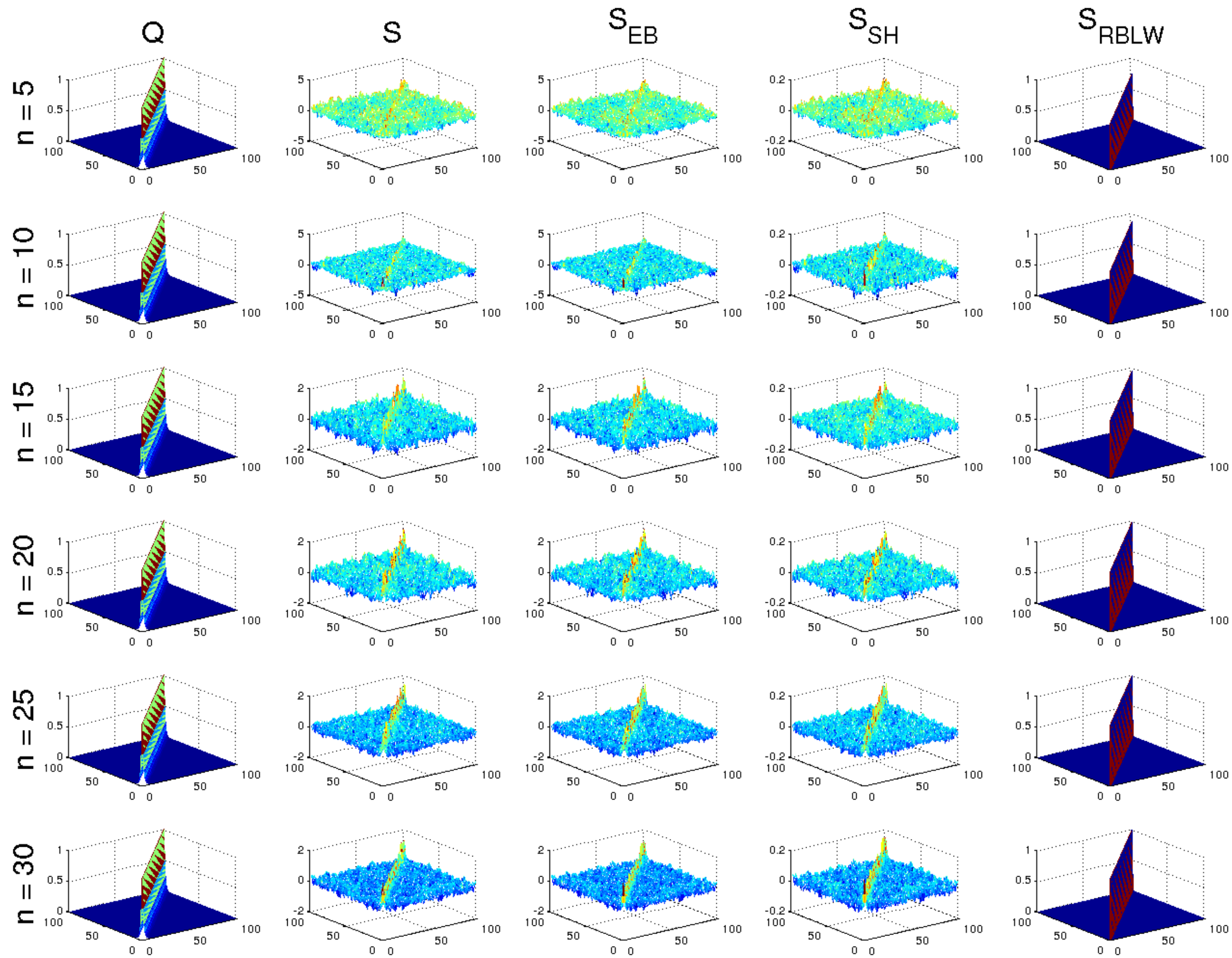
- Where the parameters are chosen so that

$$\min_{\rho_1,\rho_2} \mathbb{E}\left[\|Q - S_\Theta\|\right]$$

- Rao Blackwell Ledoit Wolf (2004) estimator

$$\widehat{\Sigma}_{RBLW} = \rho_{RBLW} \cdot I_{p\times p} + (1 - \rho_{RBLW}) \cdot S, \text{ where}$$
$$\rho_{RBLW} = \min\left(\frac{\frac{n-2}{n}\cdot \mathrm{tr}\left(S^2\right) + \mathrm{tr}^2(S)}{(n+2)\cdot\left[\mathrm{tr}(S^2) - \frac{\mathrm{tr}^2(S)}{p}\right]}, 1\right)$$

Validation on an Autoregressive process (AR)



S_{EB} is the empirical Bayesian estimator and S_{SH} is Stein's SVD decomposition-based estimator.

Stochastic ED as (dual) block-angular LPs

Extensive form

$$
\begin{array}{lccccccccccc}
\min & c_0^T x_0 & + & c_1^T x_1 & + & c_2^T x_2 & + & \ldots & + & c_N^T x_N & & \\
\text{s.t.} & A x_0 & & & & & & & & & = & b_0, \\
 & T_1 x_0 & + & W_1 x_1 & & & & & & & = & b_1, \\
 & T_2 x_0 & & & + & W_2 x_2 & & & & & = & b_2, \\
 & \vdots & & & & & & \ddots & & & & \vdots \\
 & T_N x_0 & & & & & & & + & W_N x_N & = & b_N, \\
 & \multicolumn{11}{l}{x_0 \geq 0, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad \ldots, \quad x_N \geq 0.}
\end{array}
$$

- Easy to build practical instances having billions of decision variables and constraints

  ➔ Requires distributed memory computers
- Real-time solution needed in power grid applications

Interior-point optimization solver - PIPS

Convex quadratic problem

$$\text{Min } \frac{1}{2}x^T Q x + c^T x$$
$$\text{subj. to. } Ax = b$$
$$x \geq 0$$

IPM Linear System

$$\begin{bmatrix} Q+\Lambda & A^T \\ A & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = rhs$$

2 solves per IPM iteration
- predictor direction
- corrector direction

Multi-stage SP

Two-stage SP

nested arrow-shaped linear system (modulo a permutation)

S is the number of scenarios

$$\begin{bmatrix}
H_1 & B_1^T & & & & & & 0 & 0 \\
B_1 & 0 & & & & & & A_1 & 0 \\
 & & H_2 & B_2^T & & & & 0 & 0 \\
 & & B_2 & 0 & & & & A_2 & 0 \\
 & & & & \ddots & & & \vdots & \vdots \\
 & & & & & H_S & B_N^T & 0 & 0 \\
 & & & & & B_N & 0 & A_S & 0 \\
0 & A_1^T & 0 & A_2^T & \dots & 0 & A_S^T & H_0 & A_0^T \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & A_0 & 0
\end{bmatrix}$$

Special Structure of KKT System (Arrow-shaped)

$$\begin{bmatrix} K_1 & & & B_1 \\ & \ddots & & \vdots \\ & & K_N & B_N \\ B_1^T & \dots & B_N^T & K_0 \end{bmatrix} \begin{bmatrix} \Delta z_1 \\ \vdots \\ \Delta z_N \\ \Delta z_0 \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \\ r_0 \end{bmatrix}$$

where,

$$K_i := \begin{bmatrix} \bar{Q}_i & W_i^T \\ W_i & 0 \end{bmatrix}, \qquad K_0 := \begin{bmatrix} \bar{Q} & A^T \\ A & 0 \end{bmatrix},$$

$$B_i := \begin{bmatrix} 0 & 0 \\ T_i & 0 \end{bmatrix}, \qquad i = 1, 2, \dots, N.$$

Block Elimination

$$\begin{bmatrix} K_1 & & & B_1 \\ & \ddots & & \vdots \\ & & K_N & B_N \\ B_1^T & \dots & B_N^T & K_0 \end{bmatrix} \begin{bmatrix} \Delta z_1 \\ \vdots \\ \Delta z_N \\ \Delta z_0 \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \\ r_0 \end{bmatrix}$$

Multiply row *i* by $-B_i^T K_i^{-1}$ and sum all the rows to obtain

$$\left(K_0 - \sum_{i=1}^{N} B_i^T K_i^{-1} B_i \right) \Delta z_0 = r_0 - \sum_{i=1}^{N} B_i^T K_i^{-1} r_i$$

The matrix $C := K_0 - \sum_{i=1}^{N} B_i^T K_i^{-1} B_i$ is the Schur-complement of the diagonal $K_1, \dots, K_N$ block.

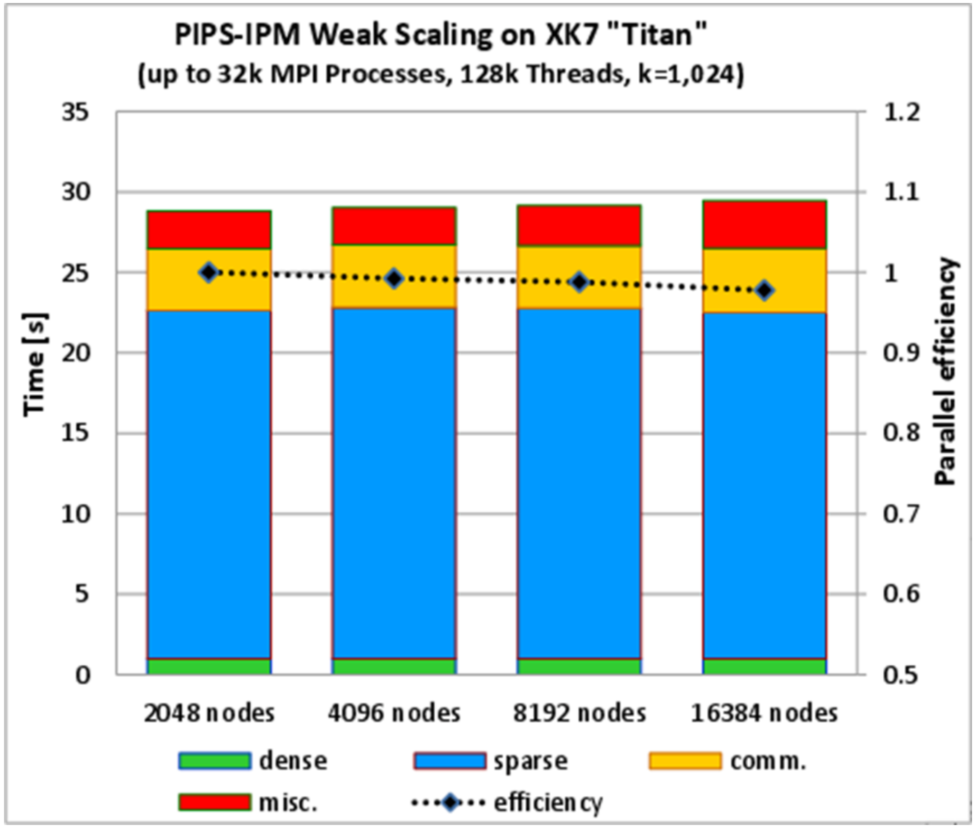
Parallel Solution Procedure for KKT System

1. Calculate $B_i^T K_i^{-1} B_i,\ i = 1, \ldots, N$ ("Compute S.C.")
2. Form $C := K_0 - \sum_{i=1}^N B_i^T K_i^{-1} B_i$ ("Form S.C.")
3. Factorize $C = L_0 D_0 L_0^T$ ("Factor S.C.")
4. Solve $\Delta z_0 = C^{-1}(r_0 - \sum_{i=1}^N B_i^T K_i^{-1} r_i)$
5. Solve $\Delta z_i = K_i^{-1}(B_i \Delta z_0 - r_i),\ i = 1, \ldots, N$

- Steps 1 and 5 trivially parallel
  - "Scenario-based decomposition"
- Extra care needed for computational bottlenecks 2, 3, and 5: multithreaded or GPU accelerated linear algebra, tuned coomunication, etc.
- Realtime is achieved using with an augmented incomplete factorization coupled with BiCGStab (to speed-up 1).

PIPS performance (Petra et al., 2013)

- C++ code, MPI+OpenMP, runs on a variety of high performance computing platforms: IBM BG/P-Q (Argonne), Cray XK7 (Oak Ridge), Cray XE6 and XC30 (Swiss National Computing Centre)



The largest instance has 4.08 billion decision variables and 4.12 billion constraints.

Simulations of State of Illinois' power grid

- The network consists of 2522 lines, 1908 buses, 870 demand buses, 225 generators, of which 32 are wind farms.
- Some of the wind farms are hypothetical and replace coal generators.
- Wind "installed" capacity is 17%. Adoption in around 15%.

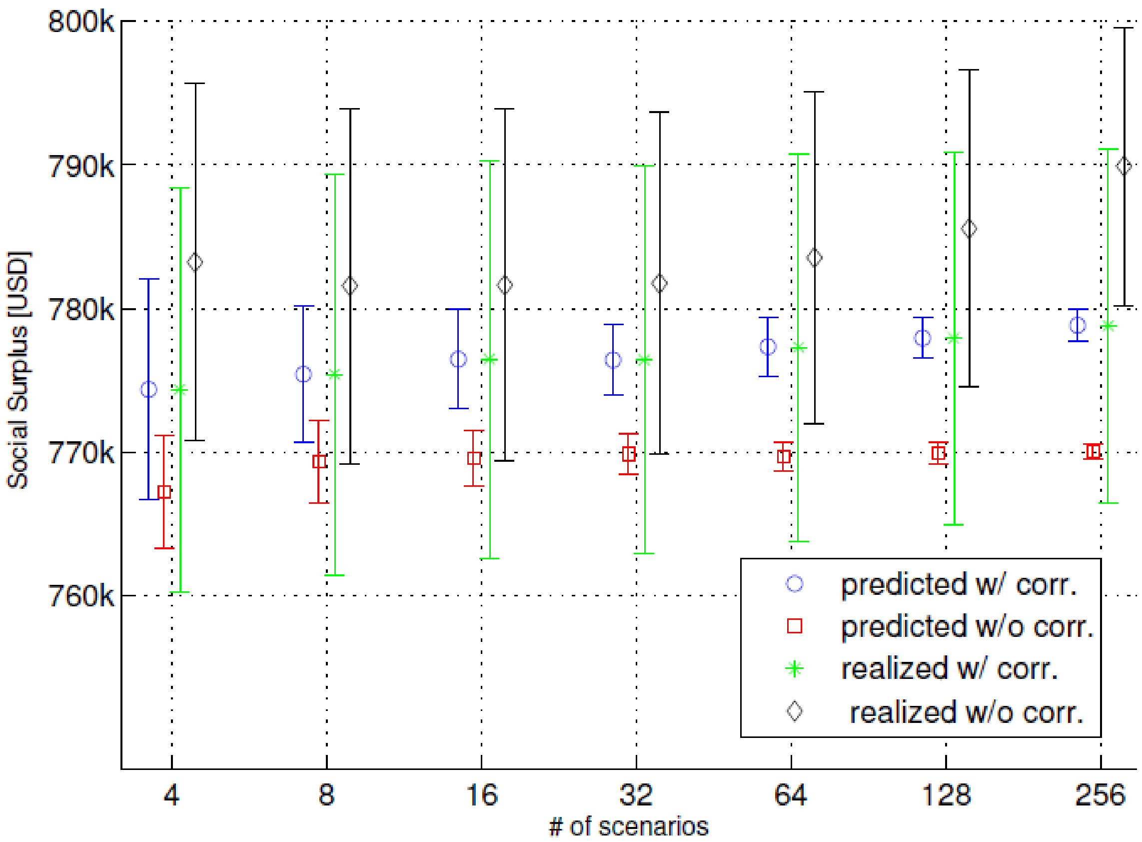
- RBLW covariance matrix ("corr.") vs diagonal covariance matrix ("indep.")
  - Dispatch cost
  - Ahead/realtime prices

- Both dispatch cost and the prices are random under the resampling scheme, therefore we compute confidence intervals.

- A problem with 256 scenarios has a little bit less than 1 million variables and 1 million constraints.

- Used Argonne's BG/P "Intrepid" and BG/Q "Mira" platforms for computing confidence intervals.

Dispatch cost – correlation vs independent resampling

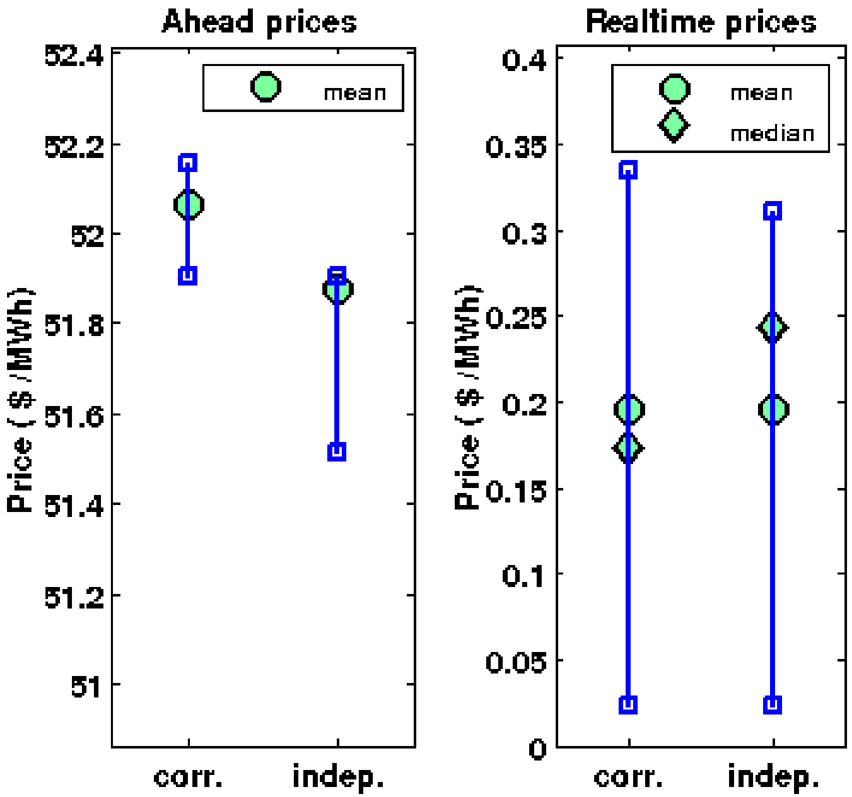


95% confidence intervals for the dispatch cost for predicted and realized costs, each with (w/) and without (w/o) correlation information

- The smallest gap, 1.42% or $10,967 that occurs for batches of 256 scenarios can potentially add up to approx. $100 million over a year.
- The gap does not seem to close as the number of scenarios increases.
- About 256 scenarios seem to offer a decent approximation (std. dev. is 0.36%)

Prices - correlation vs independent resampling

95% confidence intervals for prices at a typical bus



- As expected from the previous slide, the prices computed with correlation information are higher than the prices computed with no correlation.
- Realtime prices are about the same magnitude.
- Opportunities for market arbitrage for players with better covariance information.

Conclusions

- Improper correlation estimation leads to inefficient pricing and higher dispatch costs, negatively impacting social welfare.
- We advocate for centralized weather forecasting in power grid dispatch.
- Better covariance estimation potentially leads to more efficient pricing.

- Details in C.Petra, E.D.Nino, V.Zavala, M.Anitescu “On the correlation of wind covariance estimation in economic dispatch models”, to be submitted to IEEE Power Systems.

Thank you for your attention!

Any questions?

Additional material

Empirical Bayesian estimator

- Again, n<<p

$$S_{EB} = \frac{p \cdot n - 2 \cdot n - 2}{p \cdot n^2} \cdot m_{EB} \cdot I + \frac{n}{n+1} \cdot S \in \mathbb{R}^{p \times p}, \text{ where}$$

$$m_{EB} = tr(S \cdot I)/p$$

Stein's SVD decomposition-based estimator

$$\mathbf{S} = \mathbf{U} \cdot \Sigma \cdot \mathbf{U}^{\mathrm{t}} ,$$

$$\Sigma = \mathbf{diag} \left(\sigma_1, \sigma_2, \ldots, \sigma_p \right) \in \Re^{p \times p} .$$

$$\hat{\sigma}_i = \frac{n \cdot \sigma_i}{n - p + 1 + 2 \cdot \sigma_i \cdot \sum_{k=1, k \neq i}^{p} \frac{1}{\sigma_i - \sigma_k}} , i = 1, 2, \ldots, p ,$$

$$\Lambda = \mathbf{diag} \left(\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_p \right) \in \Re^{p \times p} ,$$

$$\mathbf{S}_{\mathrm{SH}} = \mathbf{U} \cdot \Lambda \cdot \mathbf{U}^{\mathrm{t}} \in \Re^{p \times p} .$$