Solving MPEC Models with the KNITRO Nonlinear Solver

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Outline

1. KNITRO Short Introduction
2. What Are MPECs and Why Are They Useful?
3. Application in OPF Models (faster alternative to MIP)

The KNITRO solver (in a nutshell)

KNITRO: short introduction

KNITRO is a *commercially supported* optimization toolbox with a focus on solving large-scale nonlinear optimization problems.

Developed by Ziena Optimization LLC since 2001, distributed/supported by Artelys

Key features

- Active-set and interior-point/barrier algorithms for continuous optimization
- MINLP algorithms and complementarity constraints for discrete optimization
- Parallel multi-start heuristic for global optimization of non-convex problems
- Other notable features:
  - presolver, crossover, run all algorithms in parallel, derivative approximations, fast infeasibility detection
  - tuner (coming soon in 9.0)

Supported interfaces

- C/C++, Java, Fortran, Python
- AMPL, AIMMS, GAMS
- MATLAB

Supported platforms: Windows 32-bit, 64-bit, Linux 64-bit, Mac OS X 64-bit

Widely used in academia and industry

Mathematical Programs with Equilibrium Constraints (MPEC)

Mathematical Program w/ Equilibrium Constraint (MPEC)

Math **P**rogramming with **E**quilibrium **C**onstraints
aka: Complementarity Constraints (MPCC)

$$\min f(x)$$
$$\text{s.t. } g(x) \geq 0$$
$$h(x) = 0$$
$$\boxed{0 \leq x_1 \perp x_2 \geq 0}$$

Applications:
- Strategic bidding
- Economic models
- Contact problems
- Traffic equilibrium
- Disjunctive conditions

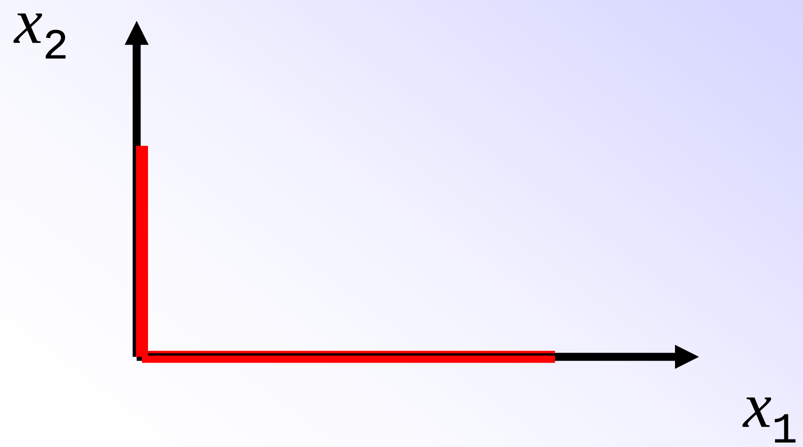
The complementarity condition means

$x_1 \geq 0$, $x_2 \geq 0$ and

either $x_1 = 0$ or $x_2 = 0$ (equivalently, $x_1 x_2 = 0$)

Why is MPEC Hard?

$$\min f(x)$$
$$\text{s.t. } g(x) \geq 0$$
$$h(x) = 0$$
$$\boxed{0 \leq x_1 \perp x_2 \geq 0}$$



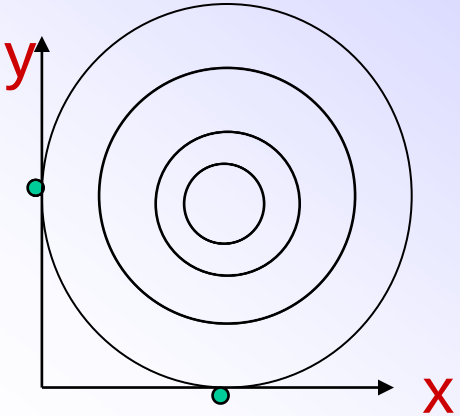
- Optimality conditions for NLP ***assume*** constraints satisfy some nice conditions (*constraint qualification*); *Constraint qualifications do not hold for MPECs*
- No interior path for interior-point methods
- Inconsistent constraint linearizations for SQP methods
- Unbounded multipliers

Why is MPEC Hard ?   (example)

$$\min \quad (x-1)^2 + (y-1)^2$$

$$\mathrm{c}_1 = x \geq 0, \qquad \mathrm{c}_2 = y \geq 0$$

$$\mathrm{c}_3 = xy = 0$$



$$\nabla c_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \nabla c_3 = \begin{pmatrix} y \\ x \end{pmatrix}$$

$$\text{When } x = 0, \quad \nabla c_3 = \alpha \nabla c_1$$

Linear dependent constraint gradients at every feasible point!

Why is MPEC Hard (in practice)?

What does this mean in practice?

- Numerical difficulties for the solver (e.g. singular linear systems)
- Challenges identifying solutions (finding multipliers to satisfy KKT conditions)

How to deal with it in practice?

- Need to identify and apply special treatment to these constraints
- *Relax*, *regularize* or *penalize* them in some way

Recent research breakthroughs allow reformulation for interior-point solvers via **penalization**

$$
\begin{aligned}
&\min f(x) \\
&\text{s.t. } g(x) \geq 0 \\
&h(x) = 0 \\
&\boxed{0 \leq x_1 \perp x_2 \geq 0}
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
&\min f(x) \boxed{+\pi\, x_1^T x_2} \\
&\text{s.t.} \quad g(x) \geq 0 \\
&h(x) = 0 \\
&x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

KNITRO MPEC Benchmarks

Comparing CPU times on a sample of MPECs

Mittelmann Benchmarks (http://plato.asu.edu/bench.html)

problem	filter	LOQO	KNITRO
bem-milanc30-s	648	f	f
frictionalblock_5	f	f	1
frictionalblock_6	f	f	f
incid-set1-32	147	56	3
incid-set1c-32	100	50	1
incid-set2-32	165	42	2
incid-set2c-32	99	f	1
pack-comp1-32	150	f	1
pack-comp1c-32	15	f	1
pack-comp1p-32	234	f	1
pack-comp2-32	301	37	1
pack-comp2p-32	186	f	1
pack-rig1-32	71	f	f
pack-rig1c-32	14	f	1

problem	filter	LOQO	KNITRO
pack-rig1p-32	64	29	1
pack-rig2-32	f	f	f
pack-rig2c-32	f	f	479
pack-rig2p-32	39	f	1
pack-rig3-32	65	f	1
pack-rig3c-32	15	f	1
siouxfls1	90	f	f
siouxfls	213	f	f

Binary variables vs MPECs

Binary variable: *x=0 or x=1* (disjunction)

NLP/MPEC form:

Let *y=x-1* and represent as $0 \leq x \perp y \geq 0$

Comparison:

- MIP vs. NLP/MPEC
- MIP gives global solution (but may be too hard to solve)
- NLP/MPEC form creates lots of non-convexity
  - Can create many local solutions (and locally infeasible points!)
  - …but can solve quickly (use as heuristic for MIP)

THE OPF PROBLEM

THE OPF MODEL

AC-OPF model

- Polar PQV formulation
- Limits on voltage magnitudes
- Maximum intensity levels on lines (nonlinear inequality constraints)
- Limits on production levels
- Kirchhoff law at each node (nonlinear equality constraints)

Generators operational constraints

- The active (P) and reactive (Q) power of a production group are
  - bounded if the group is ON
  - zeros if the group is OFF

Objective

- **Minimize losses** → similar to minimizing total production

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MINLP APPROACH

MINLP FORMULATION

- Group is OFF

$$P = 0$$

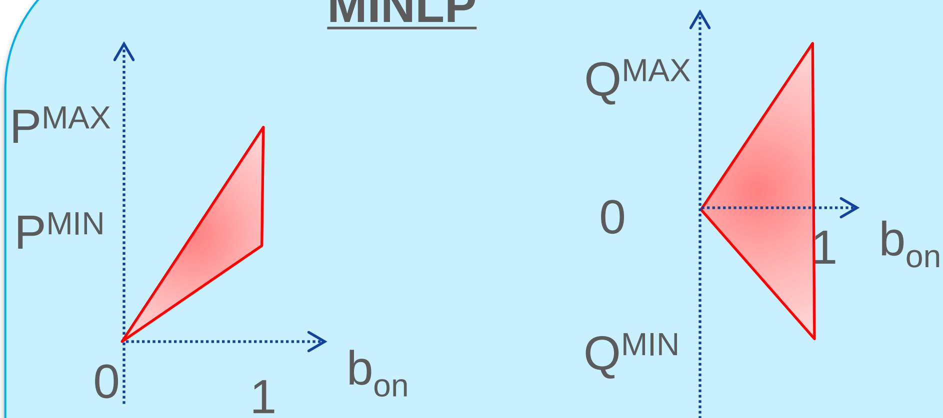
$$Q = 0$$

- Group is ON

$$P \in \left[P^{MIN}, P^{MAX}\right]$$

$$Q \in \left[Q^{MIN}, Q^{MAX}\right]$$

MINLP



$$b_{on} \in \{0,1\}$$

$$P^{MIN} \cdot b_{on} \leq P \leq P^{MAX} \cdot b_{on}$$

$$Q^{MIN} \cdot b_{on} \leq Q \leq Q^{MAX} \cdot b_{on}$$

GENERATION OF PROBLEM INSTANCES

- Use of an instance from the literature (IEEE)
  - Network composed of 662 nodes
  - Input data : topology, productions and consumptions, limits on power and voltage
  - Some phase-shifting transformers have variable transformer ratio
- Generation of 100 problem instances
  - Modifying production groups by adding a minimum capacity limit
    - Minimum capacity set to half of the maximum capacity
  - Perturbation of consumption (±50%)
    - Conservation of the total demand in active/reactive power

MINLP FORMULATION

- Algorithm : Branch-and-Bound of KNITRO 8.1.1
  - Intel Core 2 Duo @ 2.80 Ghz 4Go RAM
- Average 13.2 seconds by instance
- 100 % success rate
- Can we do faster ?

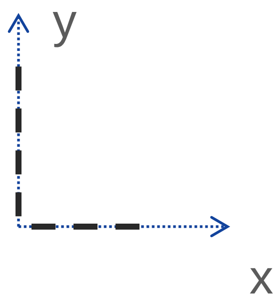
MPEC FORMULATIONS

MODELING STARTUP/SHUTDOWN OF PRODUCTION GROUP

- Use of complementarity constraints (MPEC)
  - $0 \leq x \perp y \geq 0 \Leftrightarrow (x = 0 \; or \; y = 0) \text{ and } (x, y \geq 0)$
  - Non-convex constraints on continuous variables

- Various mathematical formulations are possible:
  - $0 \leq x, y \quad xy \leq \varepsilon$
  - $0 \leq x, y$ and addition of a penalty term $\pi \cdot x \cdot y$

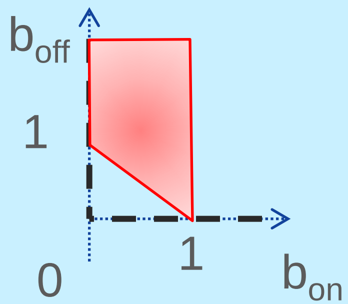
- Geometric representation:



MPEC-MINLP REFORMULATIONS

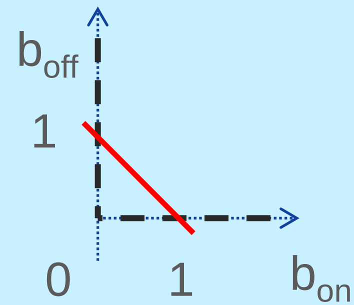
- Reformulation of $b_{on} \in \{0,1\}$ as an intersection of
  - linear polyhedron: $$P^{MIN} \cdot b_{on} \le P \le P^{MAX} \cdot b_{on}$$ $$Q^{MIN} \cdot b_{on} \le Q \le Q^{MAX} \cdot b_{on}$$
  - complementarity constraints

MPEC-MINLP1



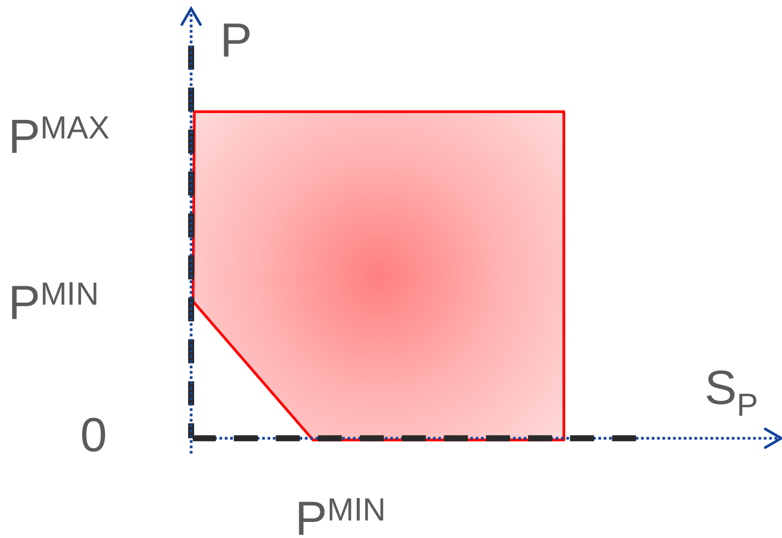
$$0 \le b_{on} \perp b_{off} \ge 0$$
$$b_{on} \le 1$$
$$b_{on} + b_{off} \ge 1$$

MPEC-MINLP2



$$0 \le b_{on} \perp 1 - b_{on} \ge 0$$

MPEC-NLP REFORMULATIONS



- Reformulation of constraints on active/reactive powers P and Q instead of b_{on}
  - Leads to MPEC constraints linking P, Q and a new continuous slack variable S_P

MPEC-NLP1

$$0 \le P + Q^2 \perp S_P \ge 0$$

$$P + S_P \ge P^{MIN}$$

$$0 \le P \le P^{MAX}$$

$$Q^{MIN} \le Q \le Q^{MAX}$$

MPEC-NLP2

$$0 \le P + Q^+ + Q^- \perp S_P \ge 0$$

$$Q = Q^+ - Q^-, Q^+ \ge 0, Q^- \ge 0$$

$$P + S_P \ge P^{MIN}$$

$$0 \le P \le P^{MAX}$$

$$Q^{MIN} \le Q \le Q^{MAX}$$

COMPUTATIONAL EXPERIMENTS

NUMERICAL RESULTS

Average results on the 100 instances

With KNITRO 8.1.1 on Intel Core 2 Duo @ 2.80 Ghz 4Go RAM

Formulation	CPU time (s)	Losses (SU)	% ON	% SUCCESS
MINLP	13.2	3.12	98.6	100
MPEC-MINLP1	1.89	5.08	80.6	80
MPEC-MINLP2	0.85	3.80	94.4	98
MPEC-NLP1	1.33	3.16	99.7	98
MPEC-NLP2	2.03	4.01	91.8	100

- The MPEC heuristics are faster than the MINLP approaches
- The various approaches converge to different solution points
  - with varying numbers of active generators
  - with varying losses

NUMERICAL RESULTS

Average results on the 100 instances

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Formulation	CPU time (s)	Losses (SU)	% ON	% SUCCESS
MINLP	13.2	3.12	98.6	100
MPEC-MINLP2	0.85	3.80	94.4	98
MPEC-NLP1	1.33	3.16	99.7	98

- MPEC-NLP1 converge towards solutions close to MINLP
  - in terms of losses (near-optimal solutions)
  - in terms of active production groups
- MPEC-MINLP2 is faster than MPEC-NLP1
  - but losses are higher (sub-optimal solutions)

CONCLUSION

- MPEC models may be a good heuristics alternative to solve non-convex MINLP problems
  - Branch-and-bound tends to be slow
  - Good when combinatorial choices are not too hard
- Some success in applying this technique to AC-OPD models
  - Use complementarity constraints to model startup/shutdown decisions
- Future work (iTesla project)
  - Apply to large instances
    - >7000 nodes, >8000 lines
  - Analyze robustness on a large number of instances

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MORE INFO AT

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