Decomposition Approaches to UC with Transmission Switching under N-1 Reliability Requirements

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The inspiration

- "Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability." Hedman, Ferris, O'Neill, Fisher, and Oren. *IEEE Trans Power Systems*. 25(2) 2010.

"Just allowing processing [of the RTS-96 test case] *at the root node typically takes 20h on a desktop workstation…"*

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“Just allowing processing [of the RTS-96 test case] *at the root node typically takes 20h on a desktop workstation…”*

“While reducing [the] optimality gap to zero is an interesting academic issue…”

The problem

- RTS-96 test case
  - 73 busses, 115 non-radial lines, 99 generators
  - 214 contingencies
- Allow switching of any non-radial line
- Explicitly enforce N-1

- Explicitly omitting
  - FTR
  - Reconnection stability / transient issues
  - Operational computation time limits

The solution: UC + Transmission Switching + *N-1*

$$\text{Minimize}: \sum_t \sum_g \left(c_g P_{g0t} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt}\right) \quad (1)$$

$$\text{S.t.} \quad \theta^{\min} \le \theta_{nct} \le \theta^{\max}, \quad \forall\, n, c, t \quad (2)$$

$$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{g0t} = d_{nt},$$
$$\forall\, n, \quad c = 0, \text{ transmission contingency states } c, t \quad (3a)$$

$$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{gct} = d_{nt},$$
$$\forall\, n, \text{ generator contingency states } c, t \quad (3b)$$

$$P_{kc}^{\min} N1_{kc} z_{kt} \le P_{kct} \le P_{kc}^{\max} N1_{kc} z_{kt}, \quad \forall\, k, c, t \quad (4)$$

$$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} + (2 - z_{kt} - N1_{kc}) M_k \ge 0, \quad \forall\, k, c, t \quad (5a)$$

$$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} - (2 - z_{kt} - N1_{kc}) M_k \le 0, \quad \forall\, k, c, t \quad (5b)$$

$$P_g^{\min} N1_{gc} u_{gt} \le P_{gct} \le P_g^{\max} N1_{gc} u_{gt}, \quad \forall\, g, c, t \quad (6)$$

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1}, \quad \forall\, g, t \quad (7)$$

$$\sum_{q=t-UT_g+1}^{t} v_{g,q} \le u_{g,t}, \quad \forall\, g, t \in \{UT_g, \ldots, T\} \quad (8)$$

$$\sum_{q=t-DT_g+1}^{t} w_{g,q} \le 1 - u_{g,t}, \forall\, g, t \in \{DT_g, \ldots, T\} \quad (9)$$

$$P_{g0t} - P_{g0,t-1} \le R_g^{+} u_{g,t-1} + R_g^{SU} v_{g,t}, \quad \forall\, g, t \quad (10)$$

$$P_{g0,t-1} - P_{g0,t} \le R_g^{-} u_{g,t} + R_g^{SD} w_{g,t}, \quad \forall\, g, t \quad (11)$$

$$P_{gct} - P_{g0,t} \le R_g^{+}, \quad \forall\, g, c, t \quad (12)$$

$$P_{g0,t} N1_{gc} - P_{gct} \le R_g^{-}, \quad \forall\, g, c, t \quad (13)$$

$$0 \le v_{g,t} \le 1, \quad \forall\, g, t \quad (14)$$

$$0 \le w_{g,t} \le 1, \quad \forall\, g, t \quad (15)$$

$$u_{g,t} \in \{0, 1\}, \quad \forall\, g, t \quad (16)$$

Hedman, et al., "Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability," *IEEE Trans Power Systems*, 25(2), pp.1052-1063, 2010

The challenge: MP is dense and subtle

Minimize :

$$\sum_t \sum_g \left(c_g P_{g0t} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt}\right)$$

S.t.

$$\theta^{\min} \le \theta_{nct} \le \theta^{\max}, \quad \forall\, n, c, t$$

$$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{g0t} = d_{nt},$$

$$\forall\, n, \quad c = 0, \text{ transmission contingency states } c, t$$

$$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{gct} = d_{nt},$$

$$\forall\, n, \text{ generator contingency states } c, t$$

$$P_{kc}^{\min} N1_{kc} z_{kt} \le P_{kct} \le P_{kc}^{\max} N1_{kc} z_{kt}, \quad \forall\, k, c, t$$

$$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} + (2 - z_{kt} - N1_{kc}) M_k \ge 0, \quad \forall\, k, c, t$$

$$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} - (2 - z_{kt} - N1_{kc}) M_k \le 0, \quad \forall\, k, c, t$$

$$P_g^{\min} N1_{gc} u_{gt} \le P_{gct} \le P_g^{\max} N1_{gc} u_{gt}, \quad \forall\, g, c, t$$

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1}, \quad \forall\, g, t$$

$$\sum_{q=t-UT_g+1}^{t} v_{g,q} \le u_{g,t}, \quad \forall\, g, t \in \{UT_g, \ldots, T\}$$

$$\sum_{q=t-DT_g+1}^{t} w_{g,q} \le 1 - u_{g,t}, \forall\, g, t \in \{DT_g, \ldots, T\}$$

$$P_{g0t} - P_{g0,t-1} \le R_g^+ u_{g,t-1} + R_g^{SU} v_{g,t}, \quad \forall\, g, t$$

$$P_{g0,t-1} - P_{g0,t} \le R_g^- u_{g,t} + R_g^{SD} w_{g,t}, \quad \forall\, g, t$$

$$P_{gct} - P_{g0,t} \le R_g^+, \quad \forall\, g, c, t$$

$$P_{g0,t} N1_{gc} - P_{gct} \le R_g^-, \quad \forall\, g, c, t$$

$$0 \le v_{g,t} \le 1, \quad \forall\, g, t$$

$$0 \le w_{g,t} \le 1, \quad \forall\, g, t$$

$$u_{g,t} \in \{0, 1\}, \quad \forall\, g, t$$

To a first approximation:

- DCOPF
- Economic dispatch
- Unit commitment
- Transmission switching
- N-1 contingency

Hedman, et al., "Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability," *IEEE Trans Power Systems*, 25(2), pp.1052-1063, 2010

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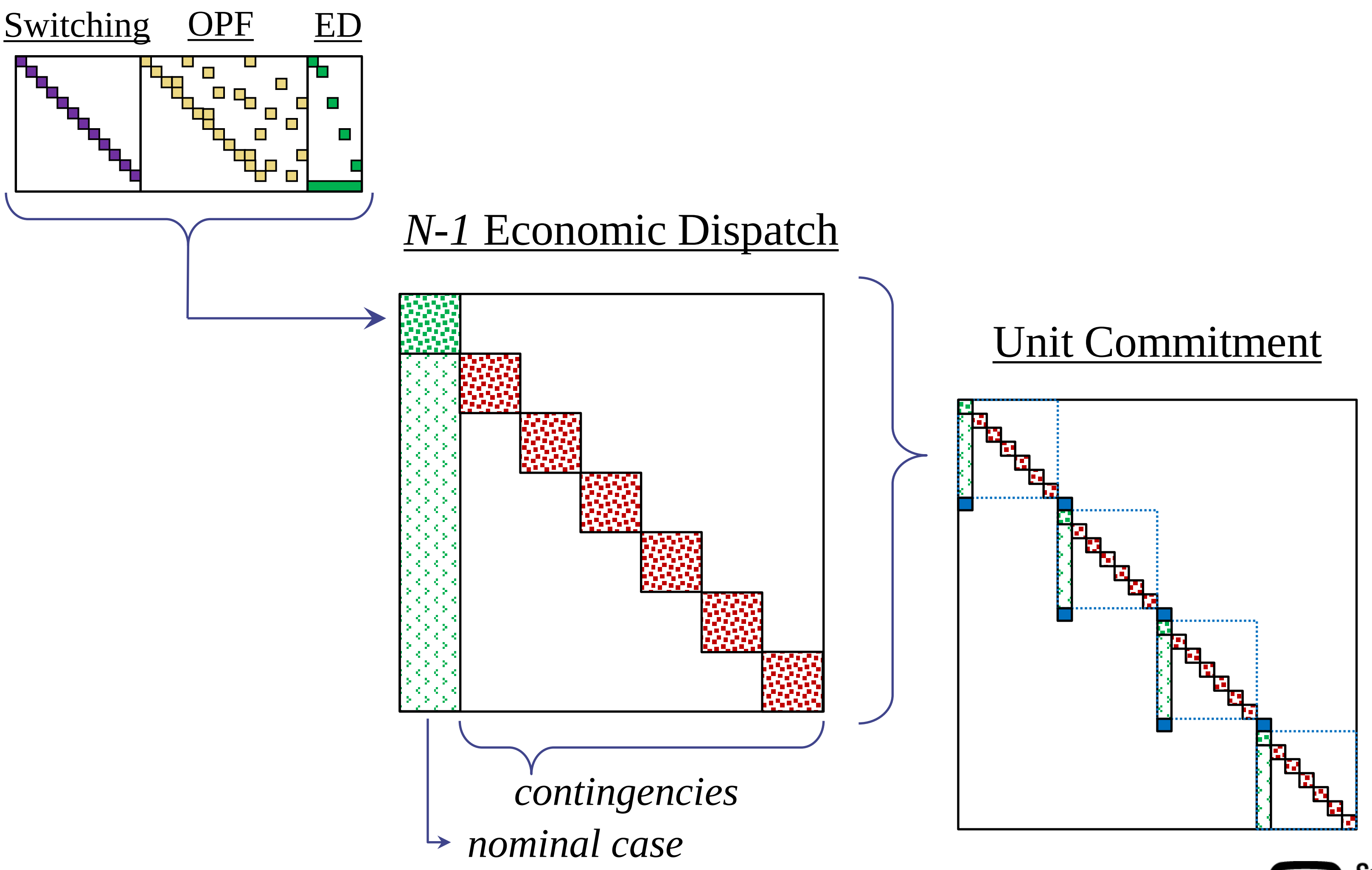
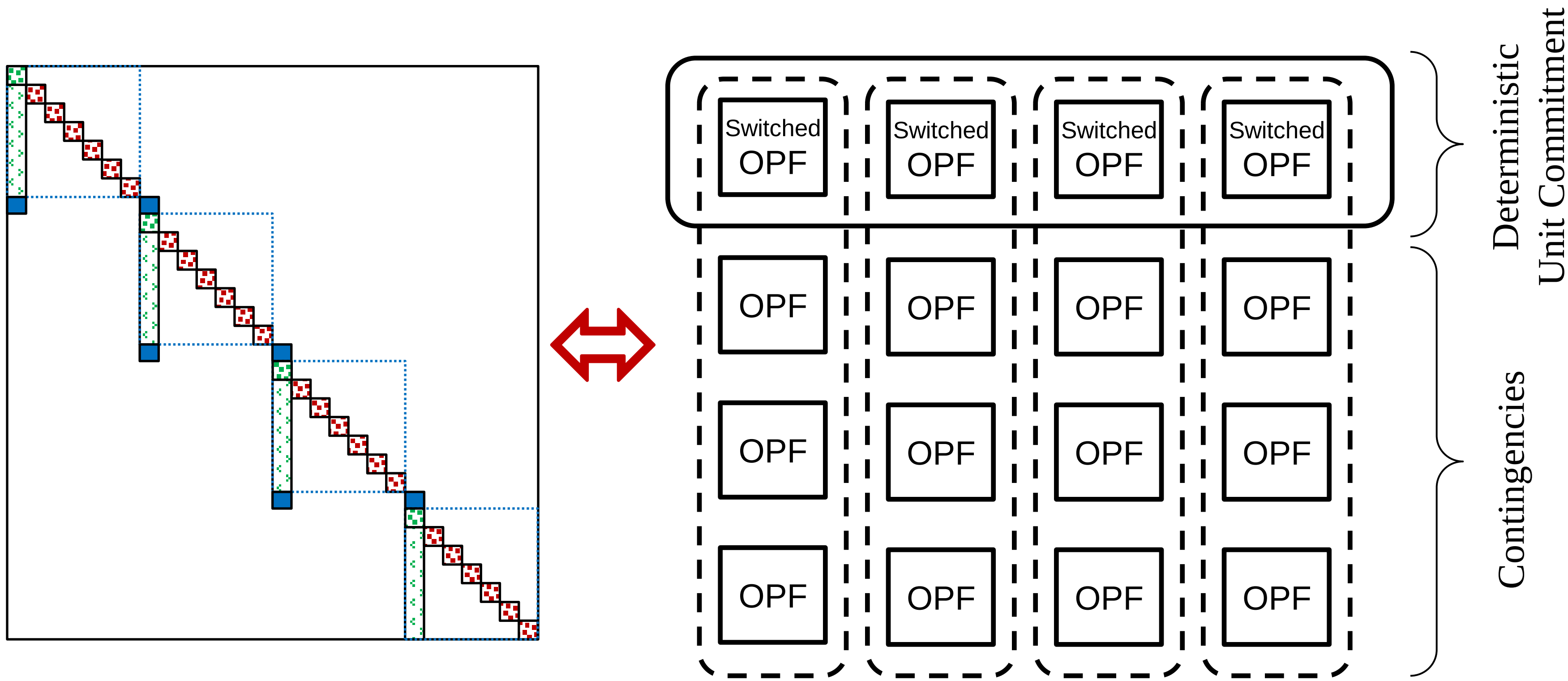
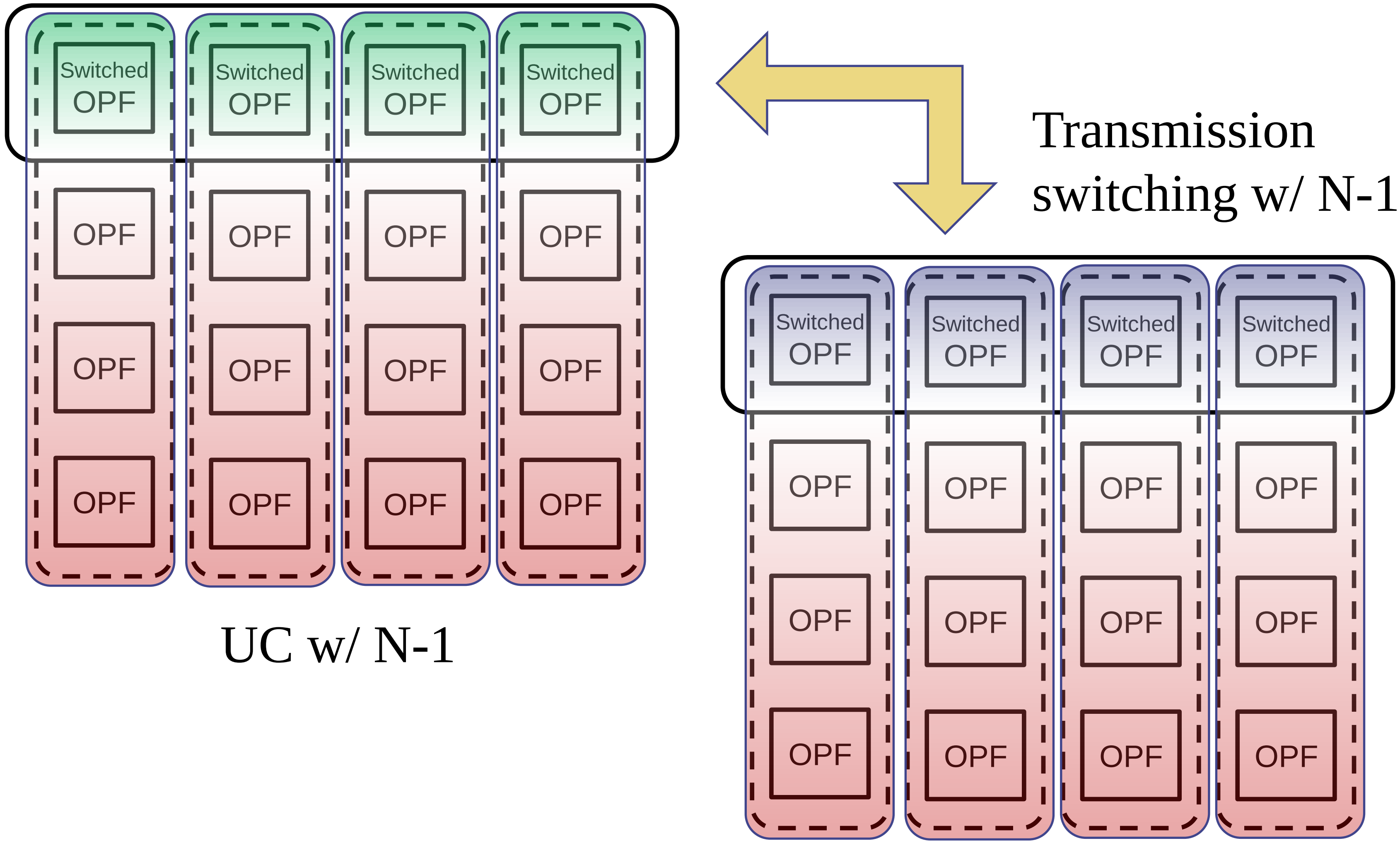
(Nonobvious) Inherent structure



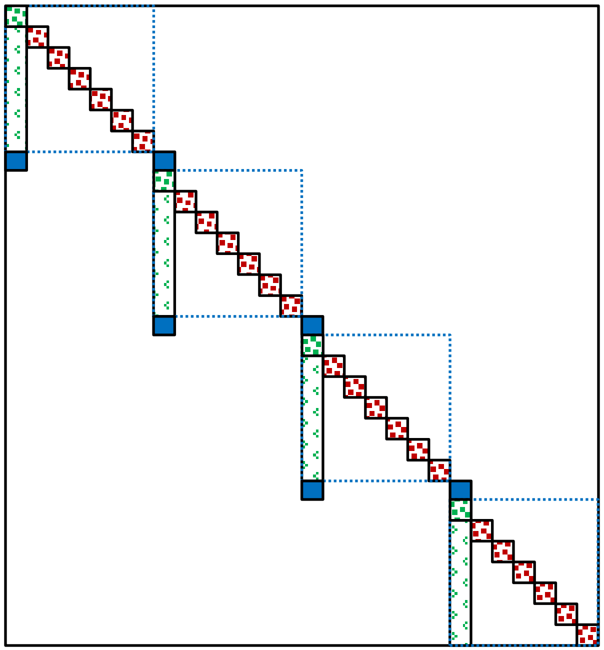
This still doesn't *quite* tell the whole story



Sequential optimization [Hedman, et al]



Block-diagonal structure



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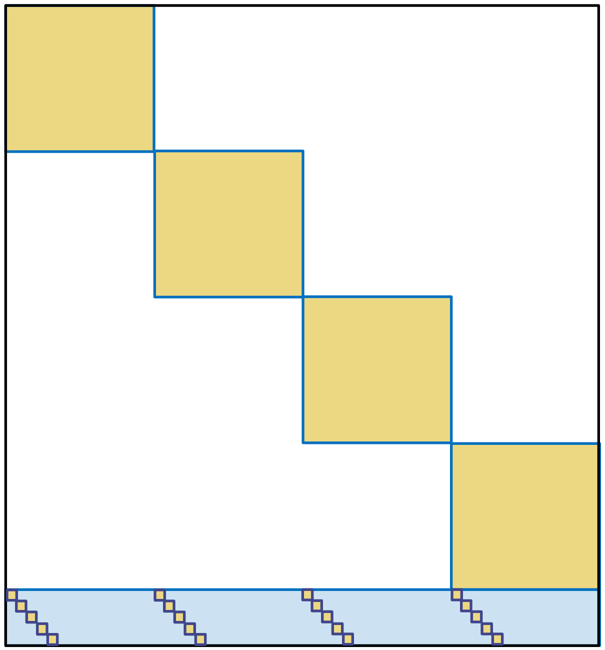


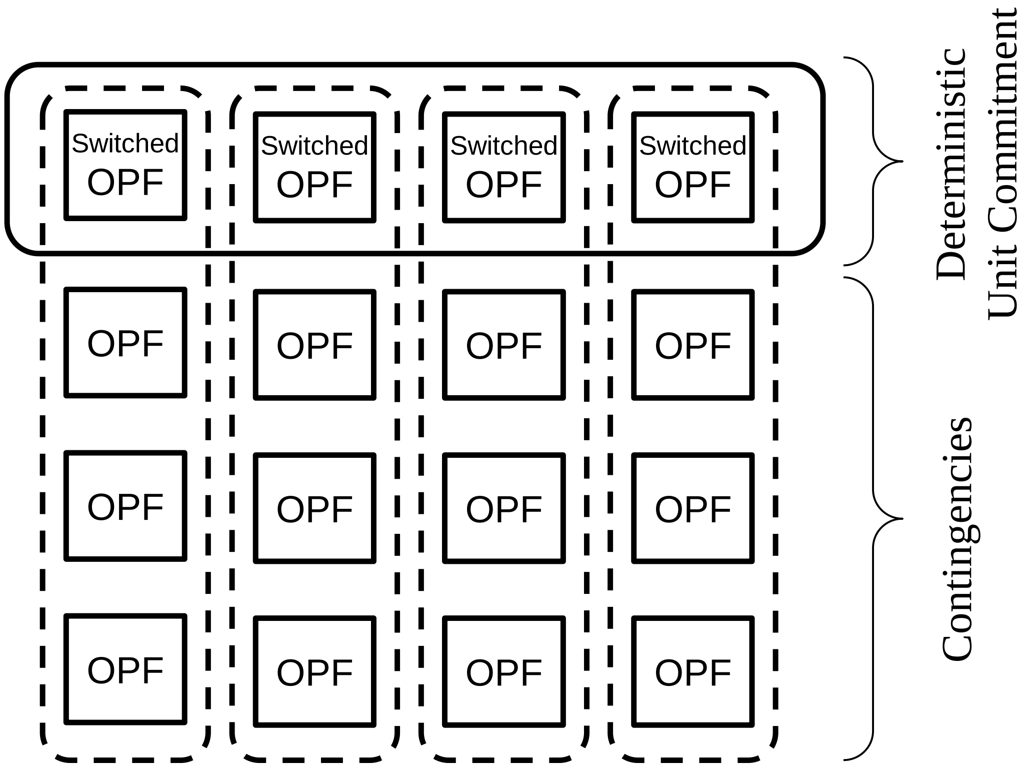
- Examples:
  - Stochastic Programming [e.g., Watson & Woodruff]
  - Parameter estimation [Word, Watson, Woodruff and Laird. 2012]

Progressive Hedging: *a Review and/or Introduction*

1. $k := 0$
2. For all $s \in \mathcal{S}$, $x_s^{(k)} := \text{argmin}_x (c \cdot x + f_s \cdot y_s) : (x, y_s) \in \mathcal{Q}_s$
3. $\bar{x}^k := (\sum_{s \in \mathcal{S}} p_s \, d_s x_s^{(k)}) / \sum_{s \in \mathcal{S}} p_s \, d_s$
4. For all $s \in \mathcal{S}$, $w_s^{(k)} := \rho(x_s^{(k)} - \bar{x}^{(k)})$
5. $k := k + 1$
6. For all $s \in \mathcal{S}$, $x_s^{(k)} := \text{argmin}_x (c \cdot x + w_s^{(k-1)} x + \rho/2 \left\| x - \bar{x}^{(k-1)} \right\|^2 + f_s \cdot y_s)$ $: (x, y_s) \in \mathcal{Q}_s$
7. $\bar{x}^{(k)} := (\sum_{s \in \mathcal{S}} p_s \, d_s x_s^{(k)}) / \sum_{s \in \mathcal{S}} p_s \, d_s$
8. For all $s \in \mathcal{S}$, $w_s^{(k)} := w_s^{(k-1)} + \rho \left(x_s^{(k)} - \bar{x}^{(k)} \right)$
9. $g^{(k)} := \frac{(1-\alpha)|\mathcal{S}|}{\sum_{s \in \mathcal{S}} p_s \, d_s} \sum_{s \in \mathcal{S}} \left\| x^{(k)} - \bar{x}^{(k)} \right\|$
10. If $g^{(k)} < \epsilon$, then go to step 5. Otherwise, terminate.

Mapping *N-1* into SP context

- What is a “scenario”?
- What are the stages / stage costs?



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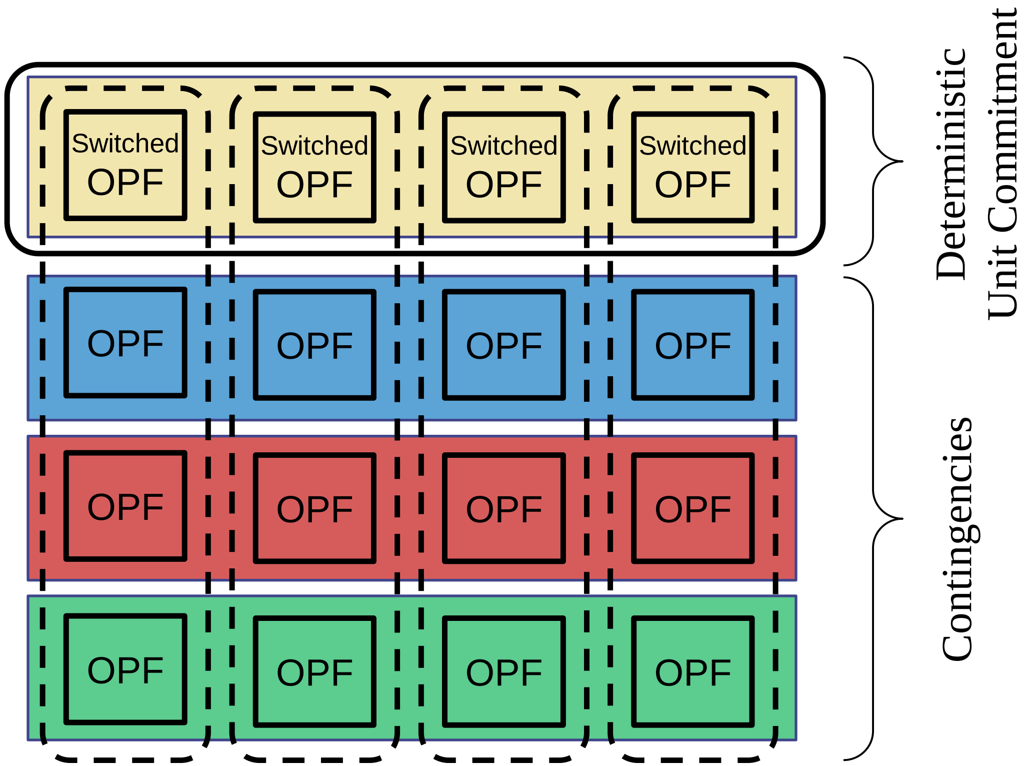

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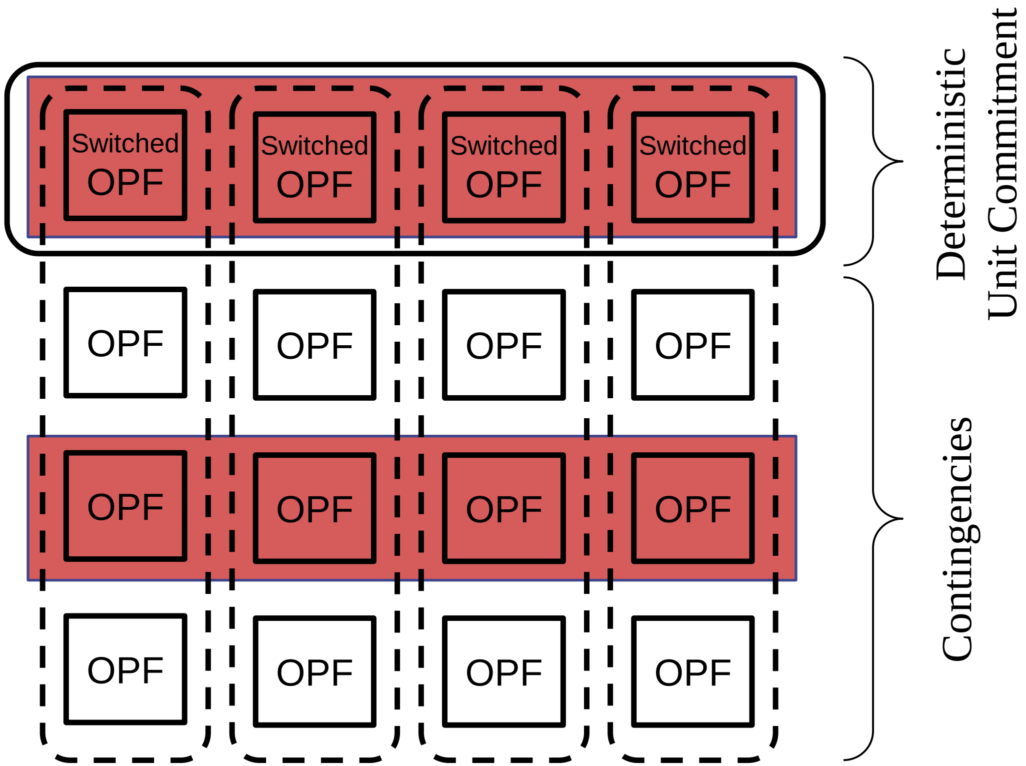
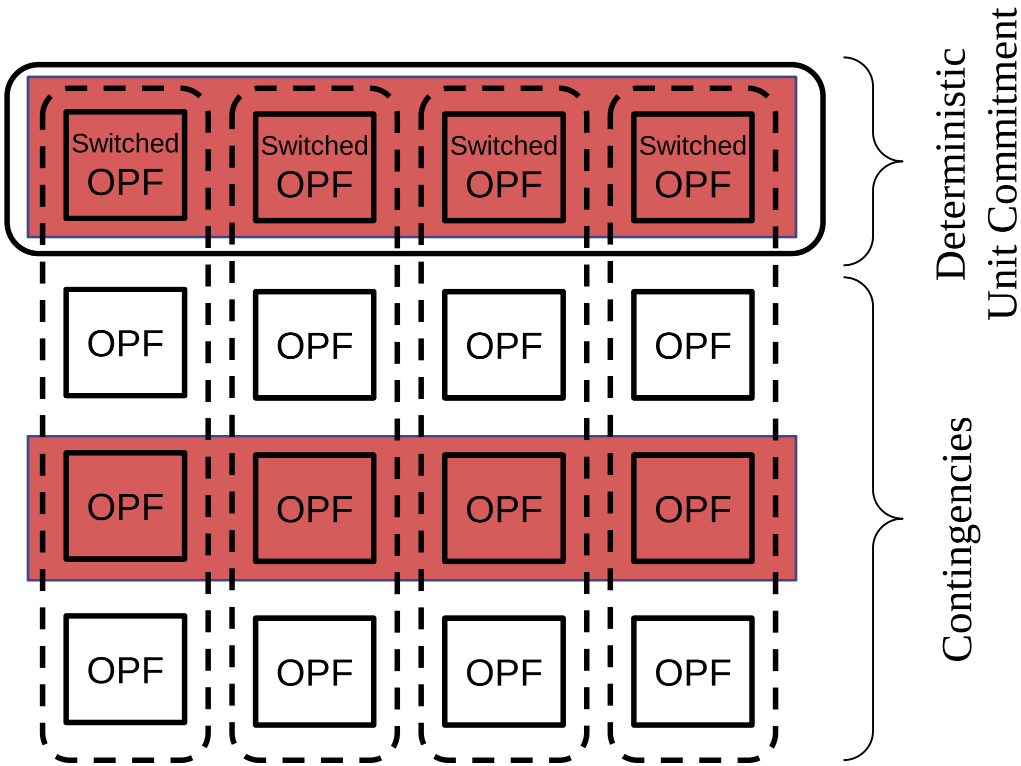


Mapping *N-1* into SP context

- What is a "scenario"?
- What are the stages / stage costs?

First stage →
(Cost = UC cost)

Second stage →
(Cost = 0)



Computational results

- Compare against the extensive form (global) solution
  - PH is a *heuristic* for MIPs
  - 5 buses, 6 non-radial lines, 7 generators

Model	EF	PH (default)
Cost	19.9756	21.1379
“EF Gap”	--	5.8%
Iterations	--	13

Computational results

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  - PH is a *heuristic* for MIPs
  - 5 buses, 6 non-radial lines, 7 generators

Model	EF	PH (default)	PH ($\rho = 0.1$)
Cost	19.9756	21.1379	20.1633
“EF Gap”	--	5.8%	0.9%
Iterations	--	13	29

A closer look at the problem

- LP bound is quite tight
- “Feasible” solutions are “easy” to find (turn units on)
- Scenario disagreement encourages over-commitment

<u>Scenarios</u>

Iter 1	19.9506	19.6689	19.6689	19.6689	19.6689	19.7923	19.6689	[…]
Iter 2	22.2871	19.6707	19.6707	19.6707	19.6707	22.6764	19.6707	[…]
Iter 3	20.9125	19.6900	19.6900	19.6900	19.6900	20.9847	19.6900	[…]
Iter 4	22.2012	19.6884	19.6884	19.6884	19.6884	22.4385	19.6884	[…]
[…]								
Iter 29	20.1633	20.1641	20.1641	20.1641	20.1641	20.1617	20.1641	[…]

- Scenario augmentation
  - Replicate critical contingencies in all scenarios
  - Heuristic pre-screening procedure: root LP bound

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Cost	19.9756	21.1379	20.1633	19.9756
"EF Gap"	--	5.8%	0.9%	0.0%
Iterations	--	13	29	1
Serial Time	**28.8**	**337**	**651**	**110**

Sub-problem solves: 0.17 - 2.49 sec;
average: 0.54

Extending to RTS-96

- From Hedman, et al.
  - N-1 UC solution: 3,245,997
  - N-1 UC w/ Switching: 3,125,185 (2 pass UC + switching)

- N-1 decomposition
  - Prescreening adds 2 common contingencies
    - LP bound: 2,886,902
  - Combined subproblems solve (to 1% gap): ~24 hours
  - Full PH solution: ??

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Alternative “decomposition”: expose structure

Minimize :

$$\sum_t \sum_g \left(c_g P_{g0t} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt}\right)$$

S.t.

$$\theta^{\min} \le \theta_{nct} \le \theta^{\max}, \quad \forall\, n, c, t$$

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$\forall\, n, \quad c = 0,$ transmission contingency states c, t

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Explicitly expose disjunctive decisions

- Transmission switching:

$$\begin{bmatrix} z_{kct} \\ P_{kct} = B_k(\theta_{k1} - \theta_{k2}) \end{bmatrix} \vee \begin{bmatrix} \neg z_{kct} \\ P_{kct} = 0 \end{bmatrix}$$

- Generation

$$\begin{bmatrix} u_{gt} \\ C_{gt} = P_{gt} c_g \\ R_g^+ \ge P_{gt} - P_{gt-1} \\ R_g^- \ge P_{gt-1} - P_{gt} \end{bmatrix} \vee \begin{bmatrix} v_{kt} \\ C_{gt} = P_{gt} c_g + c_g^{SU} \\ R_g^{SU} \ge P_{gt} - P_{gt-1} \end{bmatrix} \vee \begin{bmatrix} \neg(u_{kt} \mid v_{kt}) \\ C_{gt} = c_g^{SD} u_{kt-1} \\ R_g^{SD} \ge P_{gt-1} - P_{gt} \\ P_{gt} = 0 \end{bmatrix}$$

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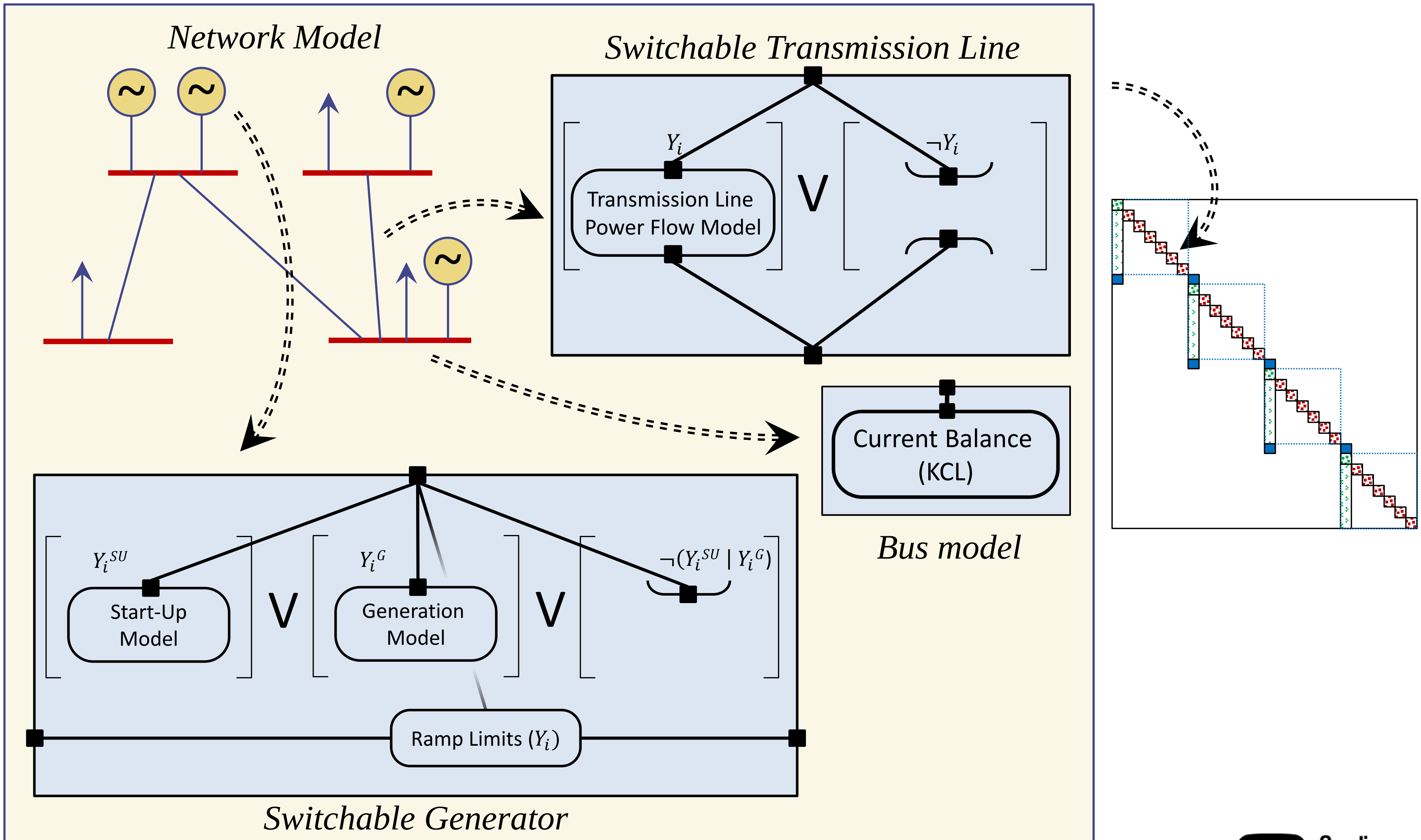
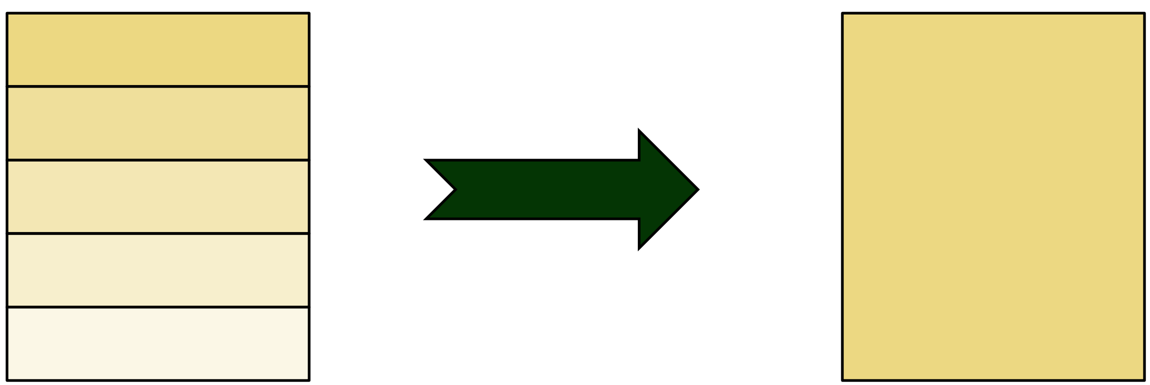
Embed within structured model



Apply standard (automatic) transformations

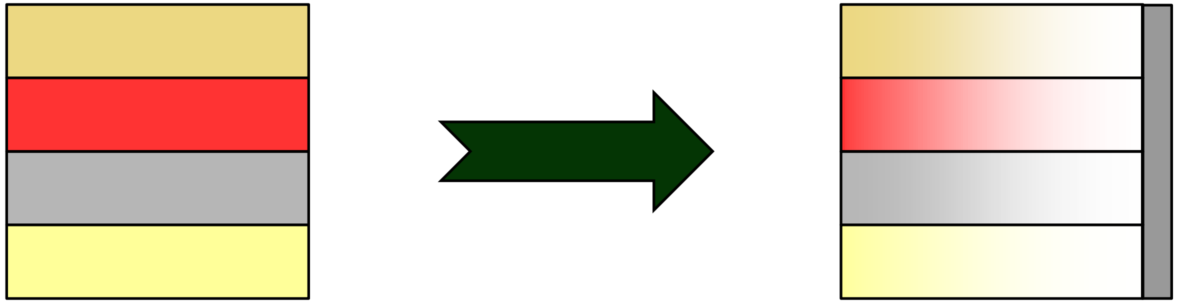
1) Construct hierarchical model
   - Generate blocks (Variables + Internal constraints)
   - "Connect" blocks by forming constraints over block connectors
2) An automatic *model transformation* "flattens" the model
   - Replicates connector constraints for each variable in connector
   - Generates aggregating constraints
   - (Eliminates redundant variables)



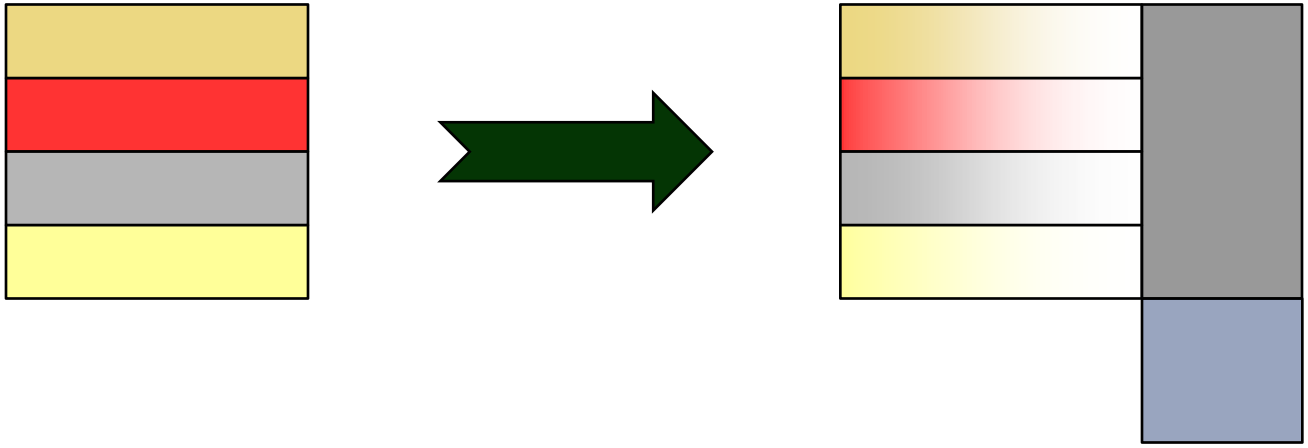
Apply standard (automatic) transformations

3) Convert disjunctions to MI(N)LP equivalents

– Big-M relaxation



– Convex hull relaxation



Optimal Solution of RTS-96

- From Hedman, et al.
  - N-1 UC solution: 3,245,997
  - N-1 UC w/ Switching: 3,125,185 (2 pass UC + switching)

	Rows	Columns	Binaries
Raw model	5,118,760	1,501,177	5,184
After presolve	2,634,851	1,062,290	4,476

- Restructured problem (complete N-1 UC w/ switching):

	Rows	Columns	Binaries
Raw model	21,232,224	13,129,692	3,796,830
After presolve	2,471,714	1,249,976	187,194

  - Solution (1e-4 gap): 2,990,004 (60,000 sec)
  - Default CPLEX settings

While “solvable,” significant cost

- Problem as posed by Hedman, et al:
  - Model generation: 201 GB; 8600 sec
  - CPLEX: ~10 GB; 60000 sec
- Allowing recourse switching in contingencies
  - Model generation: 201 GB; 8600 sec
  - CPLEX: ~356 GB; 133000 sec
- Memory limits (and current PySP design) preclude running within parallel PH

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Summary

- N-1 UC with Transmission Switching
  - Demonstrated application of SP to N-1 UC with Switching
  - Demonstrated use of structured model generation
    - Anecdote: we shouldn't try and outsmart the solvers
  - Computed optimal (1e-4) solutions for RTS-96

- Current directions
  - Parallel PH for block-based models
  - Memory reductions in core PySP
  - Demonstrate parallel PH with block-based RTS-96
  - Investigate finer contingency granularity, bundling approaches

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