Security-Constrained Optimal Power Flow with Sparsity Control and Efficient Parallel Algorithms

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X. Andy Sun (Georgia Tech, ISyE)

Joint work with Dzung T. Phan (IBM Watson Research Center)

Georgia Institute of Technology

The H. Milton Stewart School of Industrial and Systems Engineering

Outline

- SCOPF models
- Sparse solution and l_1-regularization
- Decomposition algorithms
- Computation

Security-Constrained OPF

- Preventive model:

$$
\begin{aligned}
\min_{\boldsymbol{u}_0, \boldsymbol{x}_c, \forall c \in \{0\} \cup \mathcal{C}} \quad & f(\boldsymbol{x}_0, \boldsymbol{u}_0) \\
\text{s.t.} \quad & \boldsymbol{g}_c(\boldsymbol{x}_c, \boldsymbol{u}_0) = \mathbf{0}, \quad c \in \{0\} \cup \mathcal{C} \\
& \boldsymbol{h}_c(\boldsymbol{x}_c, \boldsymbol{u}_0) \leq \mathbf{0}, \quad c \in \{0\} \cup \mathcal{C},
\end{aligned}
$$

  - $\{0\}$: Base case, no contingency
  - C: Contingency index set
  - x_0: Base-case state variables (nodal voltages)
  - x_c: Contingency-c state variable
  - u_0: Control variables (generator power output)
  - Exactly a static robust optimization model

Security-Constrained OPF

- Advantages of preventive model:
  - Robust: Control solution u_0 feasible for all contingencies
  - Easy to implement
- Disadvantages:
  - Conservative: High operational cost
  - Could be infeasible

Security-Constrained OPF

- Corrective Model:

$$
\begin{aligned}
\min_{\boldsymbol{x}_c, \boldsymbol{u}_c, \forall c \in \{0\} \cup \mathcal{C}} \quad & f(\boldsymbol{x}_0, \boldsymbol{u}_0) \\
\text{s.t.} \quad & \boldsymbol{g}_c(\boldsymbol{x}_c, \boldsymbol{u}_c) = \boldsymbol{0}, \quad c \in \overline{\mathcal{C}} \\
& \boldsymbol{h}_c(\boldsymbol{x}_c, \boldsymbol{u}_c) \le \boldsymbol{0}, \quad c \in \overline{\mathcal{C}} \\
& |\boldsymbol{u}_c - \boldsymbol{u}_0| \le \bar{\boldsymbol{u}}_c^{\max}, \quad c \in \mathcal{C}.
\end{aligned}
$$

  - u_c: Control variable for contingency-c
  - Ramping constraints: $|u_c - u_0| \le \bar{u}_c{}^{\max}$
  - Almost a two-stage robust optimization model

Security-Constrained OPF

- Advantages of corrective model:
  - Controls adaptive to contingencies
  - Achieve lower operational cost

- Disadvantage of corrective model:
  - Larger scale formulation
  - May require a large number of reschedules
    - Undesirable for system operation
    - Can we keep the flexibility of corrective model but reduce rescheduling?

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Sparse SCOPF

- Want to obtain a corrective solution with a small number of rescheduling

  ➔Sparse solution

- Still want to keep
  - similar computational complexity as the original corrective SCOPF
  - and similar cost

Security-Constrained OPF

- Recent proposal [Marano-Marcolini et al. 12']

$$
\begin{aligned}
&|u_{c,i} - u_{0,i}| \leq \bar{u}_{c,i}^{\max} s_{c,i}, \quad \forall i \in \mathcal{G}, c \in \mathcal{C} \\
&\sum_{i \in \mathcal{G}} s_{c,i} \leq N_c, \quad \forall c \in \mathcal{C} \\
&s_{c,i} \in \{0, 1\}, \quad \forall i \in \mathcal{G}, c \in \mathcal{C},
\end{aligned}
$$

  - $s_{c,i}$: Binary variable control sparsity.
  - Mixed-integer formulation: harder to solve.

SCOPF with l_1-regularization

- Our proposal:

$$
\begin{aligned}
\min_{\boldsymbol{p}^c, \boldsymbol{\theta}^c, \forall c \in \{0\} \cup \mathcal{C}} \quad & f(\boldsymbol{p}^0) + \tau \sum_{c \in \mathcal{C}} \|\boldsymbol{p}^c - \boldsymbol{p}^0\|_1 \\
\text{s.t.} \quad & \boldsymbol{p}^c - \boldsymbol{d} = \bar{\boldsymbol{B}}^c \boldsymbol{\theta}^c + \boldsymbol{p}^c_{\text{bus,shift}} + \boldsymbol{G}_{\text{sh}}, \quad \forall c \in \overline{\mathcal{C}} \\
& -f_{ij} \le \bar{B}^c_{ij}(\theta^c_i - \theta^c_j) \le f_{ij}, \forall (i,j) \in \mathcal{E}^c, c \in \overline{\mathcal{C}} \\
& \underline{\boldsymbol{p}} \le \boldsymbol{p}^c \le \bar{\boldsymbol{p}}, \quad \forall c \in \overline{\mathcal{C}} \\
& \underline{\boldsymbol{\theta}} \le \boldsymbol{\theta}^c \le \bar{\boldsymbol{\theta}}, \quad \forall c \in \overline{\mathcal{C}} \\
& |\boldsymbol{p}^c - \boldsymbol{p}^0| \le \Delta \boldsymbol{p}, \quad \forall c \in \mathcal{C},
\end{aligned}
$$

  - l_1-regularization $\sum_{c \in C} \|p^c - p^0\|_1$ induces sparsity in $(p^c - p^0)$.

SCOPF with l_1-regularization

- Reformulated l_1-model:

$$
\begin{aligned}
\min \quad & f(\boldsymbol{p}^0) + \tau \sum_{i\in\mathcal{G}, c\in\mathcal{C}} w_i^c \\
\text{s.t.} \quad & (\boldsymbol{p}^c, \boldsymbol{\theta}^c) \in \mathcal{F}_c \quad \forall c \in \overline{\mathcal{C}} \\
& -\Delta\boldsymbol{p} \le \boldsymbol{p}^c - \boldsymbol{p}^0 \le \Delta\boldsymbol{p} \quad \forall c \in \mathcal{C} \\
& -\boldsymbol{w}^c \le \boldsymbol{p}^c - \boldsymbol{p}^0 \le \boldsymbol{w}^c \quad \forall c \in \mathcal{C}.
\end{aligned}
$$

- Simplified model:

$$
\begin{aligned}
\min \quad & f(\boldsymbol{p}^0) + \tau \sum_{i\in\mathcal{G}, c\in\mathcal{C}} w_i^c \\
\text{s.t.} \quad & (\boldsymbol{p}^c, \boldsymbol{\theta}^c) \in \mathcal{F}_c \quad \forall c \in \overline{\mathcal{C}} \\
& -\Delta\boldsymbol{p} \le \boldsymbol{p}^c - \boldsymbol{p}^0 \le \Delta\boldsymbol{p} \quad \forall c \in \mathcal{C} \\
& \boldsymbol{w}^c \le \Delta\boldsymbol{p} \quad \forall c \in \mathcal{C}
\end{aligned}
$$

SCOPF with l_1-regularization

- Introduce auxiliary variables:
  - $p^{0,c} = p^0$ for all $c \in C$

- Modified l_1-model:

$$
\begin{aligned}
\min \quad & f(\boldsymbol{p}^0) + \tau \sum_{i \in \mathcal{G}, c \in \mathcal{C}} w_i^c \\
\text{s.t.} \quad & (\boldsymbol{p}^c, \boldsymbol{\theta}^c) \in \mathcal{F}_c \quad \forall c \in \overline{\mathcal{C}} \\
& -\boldsymbol{w}^c \leq \boldsymbol{p}^c - \boldsymbol{p}^{0,c} \leq \boldsymbol{w}^c \quad \forall c \in \mathcal{C} \\
& \boldsymbol{w}^c \leq \Delta \boldsymbol{p} \quad \forall c \in \mathcal{C} \\
& \boldsymbol{p}^{0,c} - \boldsymbol{p}^0 = \boldsymbol{0} \quad \forall c \in \mathcal{C}.
\end{aligned}
$$

Decomposition Method – I

- Augmented Lagrangian:

$$L_\beta = f(\boldsymbol{p}^0) + \tau \sum_{i\in\mathcal{G},c\in\mathcal{C}} w_i^c + \sum_{c\in\mathcal{C}} (\boldsymbol{\lambda}^c)^\top \left(\boldsymbol{p}^{0,c} - \boldsymbol{p}^0\right)$$

$$+ \frac{\beta}{2} \sum_{c\in\mathcal{C}} \left\|\boldsymbol{p}^{0,c} - \boldsymbol{p}^0\right\|^2,$$

  - Define $x = (p^0, \theta^0)$, $z = (z^c, \forall c \in C)$, where $z^c = (p^c, \theta^c, w^c, p^{0,c})$
  - Dual variables: $y = (\lambda^c, \forall c \in C)$
  - Therefore, $L_\beta = L_\beta(x, z, y)$
  - Augmented Lagrangian method has favorable convergence property. However, quadratic terms not decomposable.

Decomposition Method – II

- Alternating Direction Method of Multiplier (ADMM)
  - ADMM enjoys a resurgence of interest:
    - Wide applications & robust convergence
    - Deep connection to proximal algorithm & splitting principle
- ADMM:

$$
\begin{aligned}
\boldsymbol{x}_{t+1} &= \arg\min_{\boldsymbol{x}\in X} L_\beta(\boldsymbol{x}, \boldsymbol{z}_t, \boldsymbol{y}_t) \\
\boldsymbol{z}_{t+1} &= \arg\min_{\boldsymbol{z}\in Z} L_\beta(\boldsymbol{x}_{t+1}, \boldsymbol{z}, \boldsymbol{y}_t) \\
\boldsymbol{y}_{t+1} &= \boldsymbol{y}_t + \beta\left(\boldsymbol{A}\boldsymbol{x}_{t+1} + \boldsymbol{B}\boldsymbol{z}_{t+1} - \boldsymbol{h}\right),
\end{aligned}
$$

Decomposition Method – III

- ADMM solving l_1-model:
  - Solve base case, get $((\boldsymbol{p}^0)_{t+1}, (\boldsymbol{\theta}^0)_{t+1})$

$$\boldsymbol{x}_{t+1} = \arg\min_{\boldsymbol{x}\in X} L_\beta(\boldsymbol{x}, \boldsymbol{z}_t, \boldsymbol{y}_t) \Rightarrow \min_{(\boldsymbol{p}^0,\boldsymbol{\theta}^0)\in\mathcal{F}_0} f\left(\boldsymbol{p}^0\right) - \left(\boldsymbol{p}^0\right)^\top \left(\sum_{c\in\mathcal{C}} (\boldsymbol{\lambda}^c)_t\right) + \frac{\beta}{2}\sum_{c\in\mathcal{C}} \left\|\left(\boldsymbol{p}^{0,c}\right)_t - \boldsymbol{p}^0\right\|^2.$$

  - Solve each contingency $c \in C$ (in parallel):

$$\boldsymbol{z}_{t+1} = \arg\min_{\boldsymbol{z}\in Z} L_\beta(\boldsymbol{x}_{t+1}, \boldsymbol{z}, \boldsymbol{y}_t) \Rightarrow \begin{aligned} \min_{\boldsymbol{p}^c,\boldsymbol{\theta}^c,\boldsymbol{p}^{0,c},\boldsymbol{w}^c} \quad & \tau \boldsymbol{e}^\top \boldsymbol{w}^c + (\boldsymbol{\lambda}^c)_t^\top \left(\boldsymbol{p}^{0,c} - \left(\boldsymbol{p}^0\right)_{t+1}\right) \\ & + \frac{\beta}{2}\left\|\boldsymbol{p}^{0,c} - \left(\boldsymbol{p}^0\right)_{t+1}\right\|^2 \\ \text{s.t.} \quad & (\boldsymbol{p}^c, \boldsymbol{\theta}^c) \in \mathcal{F}_c \\ & -\boldsymbol{w}^c \le \boldsymbol{p}^c - \boldsymbol{p}^{0,c} \le \boldsymbol{w}^c \\ & \boldsymbol{w}^c \le \Delta \boldsymbol{p} \end{aligned}$$

  - Update multipliers $c \in C$:

$$(\boldsymbol{\lambda}^c)_{t+1} = (\boldsymbol{\lambda}^c)_t + \beta\left(\left(\boldsymbol{p}^{0,c}\right)_{t+1} - \left(\boldsymbol{p}^0\right)_{t+1}\right)$$

Decomposition Method – IV

- Accelerated ADMM [Goldstein et al. 12']

$$x_{t+1} = \arg\min_{x\in X} L_\beta(x, \hat{z}_t, \hat{y}_t)$$
$$z_{t+1} = \arg\min_{z\in Z} L_\beta(x_{t+1}, z, \hat{y}_t)$$
$$y_{t+1} = \hat{y}_t + \beta(Ax_{t+1} + Bz_{t+1} - h)$$

- Acceleration step:
  - When some residue condition is satisfied:
    - let $\alpha_{t+1} = \frac{1+\sqrt{1+4\alpha_t^2}}{2}$
    - $\hat{z}_{t+1} = z_{t+1} + \frac{\alpha_t - 1}{\alpha_{t+1}}(z_{t+1} - z_t)$
    - $\hat{y}_{t+1} = y_{t+1} + \frac{\alpha_t - 1}{\alpha_{t+1}}(y_{t+1} - y_t)$

Decomposition Method – V

- Residue:
  - Primal residue:

$$d_t = \max_c (\|(\boldsymbol{p}^c)_t - (\boldsymbol{p}^c)_{t-1}\|_\infty, \|(\boldsymbol{\theta}^c)_t - (\boldsymbol{\theta}^c)_{t-1}\|_\infty,$$
$$\|(\boldsymbol{p}^{0,c})_t - (\boldsymbol{p}^{0,c})_{t-1}\|_\infty, \|(\boldsymbol{w}^c)_t - (\boldsymbol{w}^c)_{t-1}\|_\infty).$$

  - Dual residue:

$$r_t = \max_c \|(\boldsymbol{p}^{0,c})_t - (\boldsymbol{p}^0)_t\|_\infty$$

- If residue is reduced, i.e.

$$\max(d_{t+1}, r_{t+1}) < \max(d_t, r_t)$$

Take acceleration step

Computation Experiment – I

- Illustration on NE39:
  - 39 buses
  - 10 generators
  - 46 lines
  - 30 contingencies

- Compare traditional corrective model and l_1-model

Computation Experiment – II

- NE39 Traditional corrective model:

$\mathcal{G}$	Normal	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
G1	83.2	**158.0**	**148.4**	**151.0**	**166.4**	**159.6**
G2	646.0	**638.8**	**628.5**	**629.2**	646.0	**638.0**
G3	636.1	**601.2**	**666.7**	**667.5**	**578.1**	**685.5**
G4	648.4	**635.5**	**631.8**	**631.7**	**630.7**	**643.7**
G5	508.0	**493.6**	**491.2**	**491.2**	**490.6**	**499.8**
G6	642.9	**662.8**	**655.1**	**655.0**	**653.0**	**587.9**
G7	580.0	**564.7**	**561.8**	**561.8**	**560.4**	**533.6**
G8	564.0	**553.6**	**545.9**	**543.8**	**564.0**	**555.7**
G9	865.0	**853.3**	**843.1**	**839.1**	865.0	**857.2**
G10	1080.7	**1093.1**	**1081.7**	**1083.9**	**1100.0**	**1093.2**

- NE39 l_1-regularized corrective model:

$\mathcal{G}$	Normal	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
G1	83.2	**95.4**	83.2	**96.0**	**123.5**	83.2
G2	646.0	646.0	646.0	646.0	646.0	646.0
G3	624.9	**604.1**	624.9	**600.0**	**576.0**	624.9
G4	650.8	650.8	650.8	**651.4**	650.8	650.8
G5	508.0	508.0	508.0	508.0	508.0	508.0
G6	640.8	640.8	640.8	**648.9**	640.8	640.8
G7	580.0	580.0	580.0	580.0	580.0	580.0
G8	564.0	564.0	564.0	564.0	564.0	564.0
G9	865.0	865.0	865.0	865.0	865.0	865.0
G10	1055.0	**1100.0**	1055.0	**1095.1**	**1100.0**	1055.0

Computation Experiment – III

- Test systems:

Case	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{G}\rvert$	$\lvert\mathcal{B}\rvert$	$\lvert\mathcal{C}\rvert$	#Var
NE39	39	10	46	30	2728
IEEE118	118	54	186	115	32372
IEEE300	300	69	411	89	45492
PL2383	2383	320	3572	36	123051
PL3012	3012	293	3572	36	143381

- Preventive vs Corrective vs l_1-Model

		Preventive		Corrective			ℓ_1 model		
Case	$\lvert\mathcal{C}\rvert$	Cost	Time	%Res	Cost	Time	%Res	Cost	Time
NE39	30	infeas	-	98.70	516.14	1.93	7.41	516.14	1.82
IEEE118	115	1271.18	3.74	98.16	1261.46	4.74	1.83	1261.46	6.07
IEEE300	89	infeas	-	96.77	7115.87	7.11	7.75	7116.04	11.70
PL2383	36	19134.51	109.85	56.04	18461.95	119.79	0.87	18461.95	52.05
PL3012	36	25151.41	938.62	81.96	25058.10	473.67	2.15	25058.24	63.04

		ℓ_1 model by Alg. 2
Case	$\lvert\mathcal{C}\rvert$	Time
NE39	30	0.54
IEEE118	115	0.89
IEEE300	89	1.75
PL2383	36	23.04
PL3012	36	26.36

Computation Experiment – IV

- Test systems with more contingencies:

Case	$\|\mathcal{N}\|$	$\|\mathcal{G}\|$	$\|\mathcal{B}\|$	$\|\mathcal{C}\|$	#Var
NE39	39	10	46	30	2728
IEEE118	118	54	186	130	37990
IEEE300	300	69	411	120	80949
PL2383	2383	320	3572	120	615406
PL3012	3012	293	3572	150	953867

- Algorithms running times:

			SDPT3	Algorithm 1		Algorithm 2	
Case	$\|\mathcal{C}\|$	%Res	Time	Iter	Time	Iter	Time
NE39	30	9.33	2.28	179	0.72	114	**0.53**
IEEE118	130	1.64	8.07	183	2.31	44	**0.85**
IEEE300	120	9.67	17.62	228	8.55	34	**2.58**
PL2383	120	0.14	220.23	139	152.09	42	**101.81**
PL3012	150	0.04	343.45	29	**114.66**	26	143.05

  - Reported are Alg 1,2 serial run time

Conclusion

- We propose a new SCOPF model which induces sparse corrective actions
- We study decomposition methods that work well for large scale problems
- Computation shows promising performance

THANKS!