An Affine Arithmetic Method to Solve Stochastic Optimal Power Flow Problems

Mehrdad Pirnia, Claudio Canizares,
Kankar Bhattacharya, Alfredo Vaccaro

University of Waterloo

Research funded by:

Outline

- Introduction
  - Motivation
  - Research Objective
- Background Review
  - Self Validated Computation Method
  - Interval Arithmetic
  - Affine Arithmetic
    - Affine Operations
    - Non-Affine Operations
- AA-based Optimal Power Flow
  - AA-based Mathematical Model
  - Contraction Method
- Results and Discussions
  - 30-bus Benchmark System
  - 1211-bus system

Motivation

- Increased focus on renewable generation and many concerns in planning and operation of modern power systems
- Margins of operations for thermal generators to provide system reliability and efficiency
- A wide variety of probabilistic methods to incorporate uncertainties due to renewable sources integration
  - Monte-Carlo Simulation (MCS)
  - Analytical Methods (Convolution methods)
  - Probabilistic Methods (PDF)
  - Self Validated Computation Methods (SVC)
    - Interval Arithmetic (IA)
    - Affine Arithmetic (AA)

Research Objectives

- Develop an accurate and efficient AA-based OPF model to incorporate uncertainties in power systems.
  - Validate the AA-based operation system models with the MCS based method.
- Use the resulting AA based intervals to estimate the spinning reserve requirements in the presence of high DG penetration without PDFs.
- Test and validate the model on small and large systems.

Self-Validated Computation Method

- Most of the uncertainty analysis techniques, such as the MCS method, only capture external uncertainties.

- SVC methods keep track of internal errors inherently.

SVC Methods: Interval Arithmetic (IA)

- Considers internal and external errors, and provids the most conservative bounds

$$\hat{x} + \hat{y} = [\,\underline{x} + \underline{y}\,, \bar{x} + \bar{y}\,]$$

$$\hat{x} - \hat{y} = [\,\underline{x} - \bar{y}\,, \bar{x} - \underline{y}\,]$$

$$\hat{x}\,.\,\hat{y} = \left[\min\left\{\underline{x}.\underline{y}\,, \underline{x}.\bar{y}, \bar{x}.\underline{y}\,, \bar{x}.\bar{y}\right\}, \max\left\{\underline{x}.\underline{y}\,, \underline{x}.\bar{y}, \bar{x}.\underline{y}\,, \bar{x}.\bar{y}\right\}\right]$$

$$\frac{\hat{x}}{\hat{y}} = [\,\underline{x}\,, \bar{x}].[\,{}^{1}/_{\bar{y}}\,, {}^{1}/_{\underline{y}}]$$

- Disadvantages of IA:
  - Dependency problem
  - Overflow problem
  - Error explosion

SVC Methods: Affine Arithmetic (AA)

- It is an enhanced model for self validated numerical modeling, in which the quantities of interests presented as affine forms of certain primitive variables.
- It keeps track of correlations between computed and input quantities
- Affine representation of a value:

$$\tilde{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n$$

- Interval range:

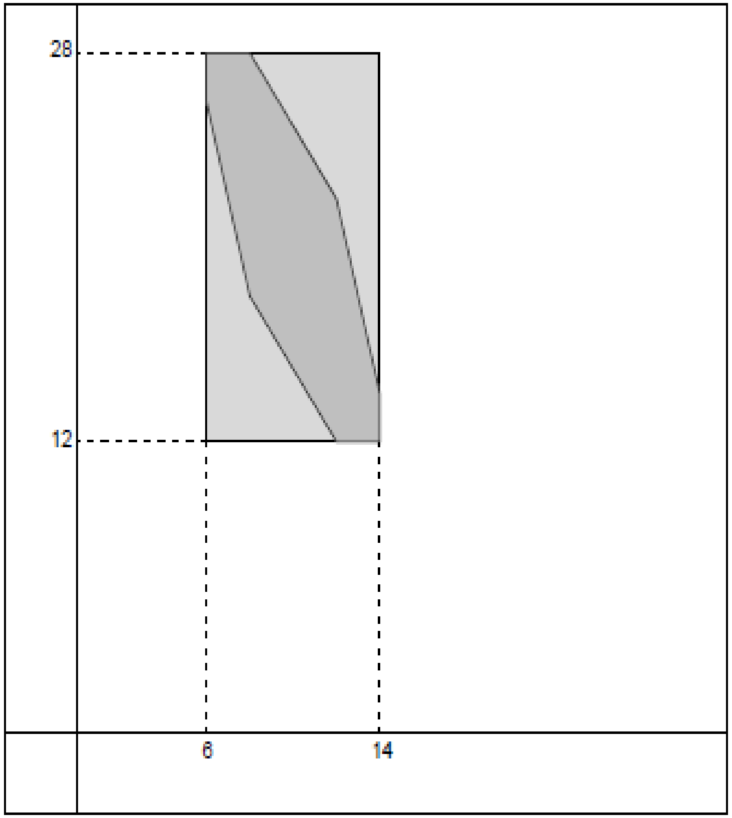
$$[\tilde{x}] = \left[x_0 - \sum_i |x_i| \,, x_0 + \sum_i |x_i|\right]$$

AA vs. IA

$$\tilde{x} = 10 + 2\varepsilon_1 + 1\varepsilon_2 \qquad - 1\varepsilon_4$$
$$\tilde{y} = 20 - 3\varepsilon_1 \qquad + 1\varepsilon_3 + 1\varepsilon_4$$

$$\hat{x} = [6\,, 14]$$

$$\hat{y} = [12\,, 28]$$



Basic Affine Operations

$$z = f(x, y) \rightarrow \tilde{z} = \tilde{f}(\tilde{x}, \tilde{y})$$

$$\tilde{x} \pm \tilde{y} = (x_0 \pm y_0) + (x_1 \pm y_1)\varepsilon_1 + \cdots + (x_n \pm y_n)\varepsilon_n$$

$$\alpha\tilde{x} = (\alpha x_0) + (\alpha x_1)\varepsilon_1 + (\alpha x_2)\varepsilon_2 + \cdots + (\alpha x_n)\varepsilon_n$$

$$\tilde{x} \pm \varphi = (x_0 \pm \varphi) + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n$$

Non-Affine Operations

$$z = f(x, y) \rightarrow \tilde{z} = f(\tilde{x}, \tilde{y})$$

$$f^*(\varepsilon_1, \dots, \varepsilon_n) = x_0 y_0 + \sum_{i=1}^{n} (x_0\, y_i + y_0\, x_i)\, \varepsilon_i + \sum_{i=1}^{n} x_i \varepsilon_i \sum_{i=1}^{n} y_i \varepsilon_i$$

$$\tilde{x}\tilde{y} = x_0\, y_0 + \sum_{i=1}^{n} (x_0\, y_i + y_0\, x_i)\, \varepsilon_i + z_k\, \varepsilon_k$$

$$z_k = \sum_{i=1}^{n} |x_i| \sum_{i=1}^{n} |y_i|$$

AA-based OPF model

$$\min \quad F(\tilde{P}^G) = \sum_{i \in Gas} \alpha_i {\tilde{P}_i^G}^2 + \beta_i \tilde{P}_i^G + c_i$$

s.t.:

$$\Delta\tilde{P}_i\left(\tilde{e}_i, \tilde{f}_i, \tilde{I}_{r\,i}, \tilde{I}_{im\,i}, \tilde{P}_i^G, \tilde{P}_i^D\right) = 0 \qquad \forall i \in N$$

$$\Delta\tilde{Q}_i\left(\tilde{e}_i, \tilde{f}_i, \tilde{I}_{r\,i}, \tilde{I}_{im\,i}, \tilde{Q}_i^G, \tilde{Q}_i^D\right) = 0 \qquad \forall i \in N$$

$$|\tilde{V}_i|^2 = \tilde{e}_i^2 + \tilde{f}_i^2 \qquad \forall i \in N$$

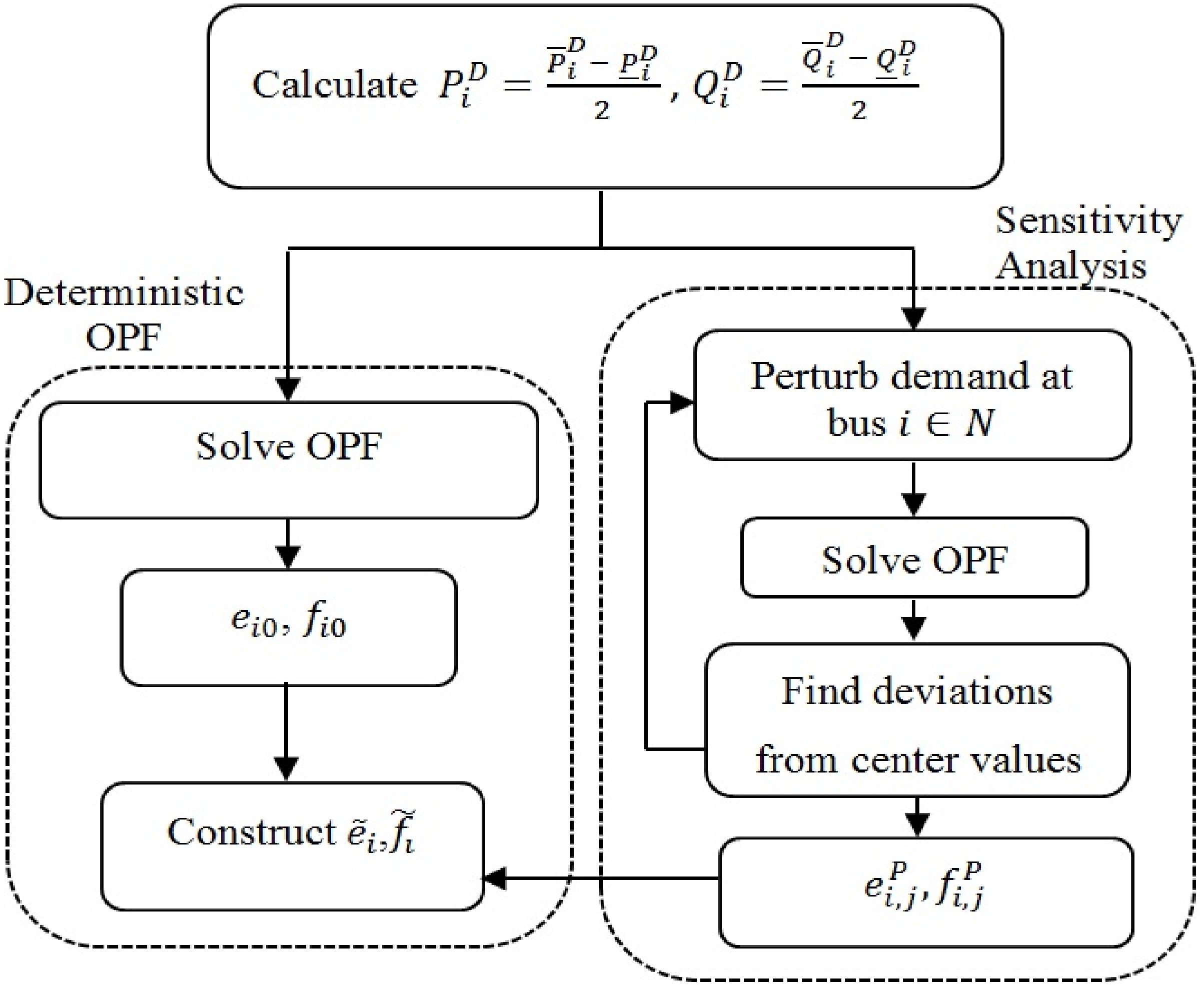
$$P_i^{min} \le \tilde{P}_i^G \le P_i^{max} \qquad \forall i \in NPG$$

$$Q_i^{min} \le \tilde{Q}_i^G \le Q_i^{max} \qquad \forall i \in NPG$$

$$I_{ij}^{min} \le \tilde{I}_{ij} \le I_{ij}^{max} \qquad \forall ij \in L$$

$$V_i^{min} \le \left|\tilde{V}_i\right| \le V_i^{max} \qquad \forall i \in N$$

AA-based OPF model



Bus Voltage Components in AA Form

$$\tilde{e}_i = e_{i0} + \sum_{j \epsilon N} e_{i,j}^{P}\, \varepsilon_{P_j^D} + \sum_{j \epsilon N} e_{i,j}^{Q}\, \varepsilon_{Q_j^D} \qquad \forall i \in N$$

$$\tilde{f}_i = f_{i0} + \sum_{j \epsilon N} f_{i,j}^{P}\, \varepsilon_{P_j^D} + \sum_{j \epsilon N} f_{i,j}^{Q}\, \varepsilon_{Q_j^D} \qquad \forall i \in N$$

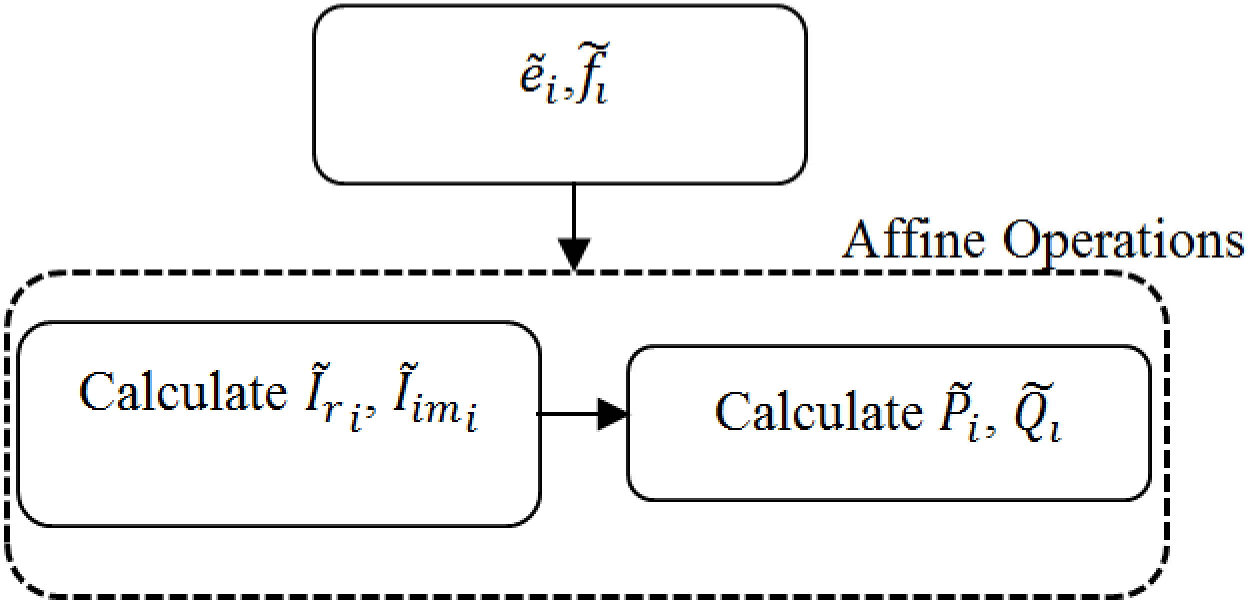
$$e_{i,j}^{P} = \left.\frac{\partial e_i}{\partial P_j^D}\right|_0 \approx \frac{e_i^N - e_i^0}{\Delta P_j^D} \qquad \forall i,j \in N$$

$$e_{i,j}^{Q} = \left.\frac{\partial e_i}{\partial Q_j^D}\right|_0 \approx \frac{e_i^N - e_i^0}{\Delta Q_j^D} \qquad \forall i,j \in N$$

$$f_{i,j}^{P} = \left.\frac{\partial f_i}{\partial P_j^D}\right|_0 \approx \frac{f_i^N - f_i^0}{\Delta P_j^D} \qquad \forall i,j \in N$$

$$f_{i,j}^{Q} = \left.\frac{\partial f_i}{\partial Q_j^D}\right|_0 \approx \frac{f_i^N - f_i^0}{\Delta Q_j^D} \qquad \forall i,j \in N$$

Affine Real and Reactive Power



Affine Real and Reactive Power

Reactive power $\tilde{P}$and $\tilde{Q}$ are calculated as follows:

$$\tilde{P} = \tilde{e}\,\tilde{I}_r + \tilde{f}\,\tilde{I}_{im}$$

$$\tilde{Q} = \tilde{f}\,\tilde{I}_r - \tilde{e}\,\tilde{I}_{im}$$

$\widetilde{P_i}$ and $\widetilde{Q_i}$ have the following affine forms:

$$\widetilde{P_i} = P_{i,0} + \sum_j P_{i,j}^{P} \varepsilon_{P_j^D} + \sum_j P_{i,j}^{Q} \varepsilon_{Q_j^D} + P_i^T \varepsilon_{T_i}$$

$$\widetilde{Q_i} = Q_{i,0} + \sum_j Q_{i,j}^{P} \varepsilon_{P_j^D} + \sum_j Q_{i,j}^{Q} \varepsilon_{Q_j^D} + Q_i^T \varepsilon_{T_i}$$

Contraction Method

$$\min \quad \tilde{F}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right)$$

$$\text{s.t.:} \quad \Delta\tilde{P}_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = 0 \qquad \forall i \in N$$

$$\Delta\tilde{Q}_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) = 0 \qquad \forall i \in N$$

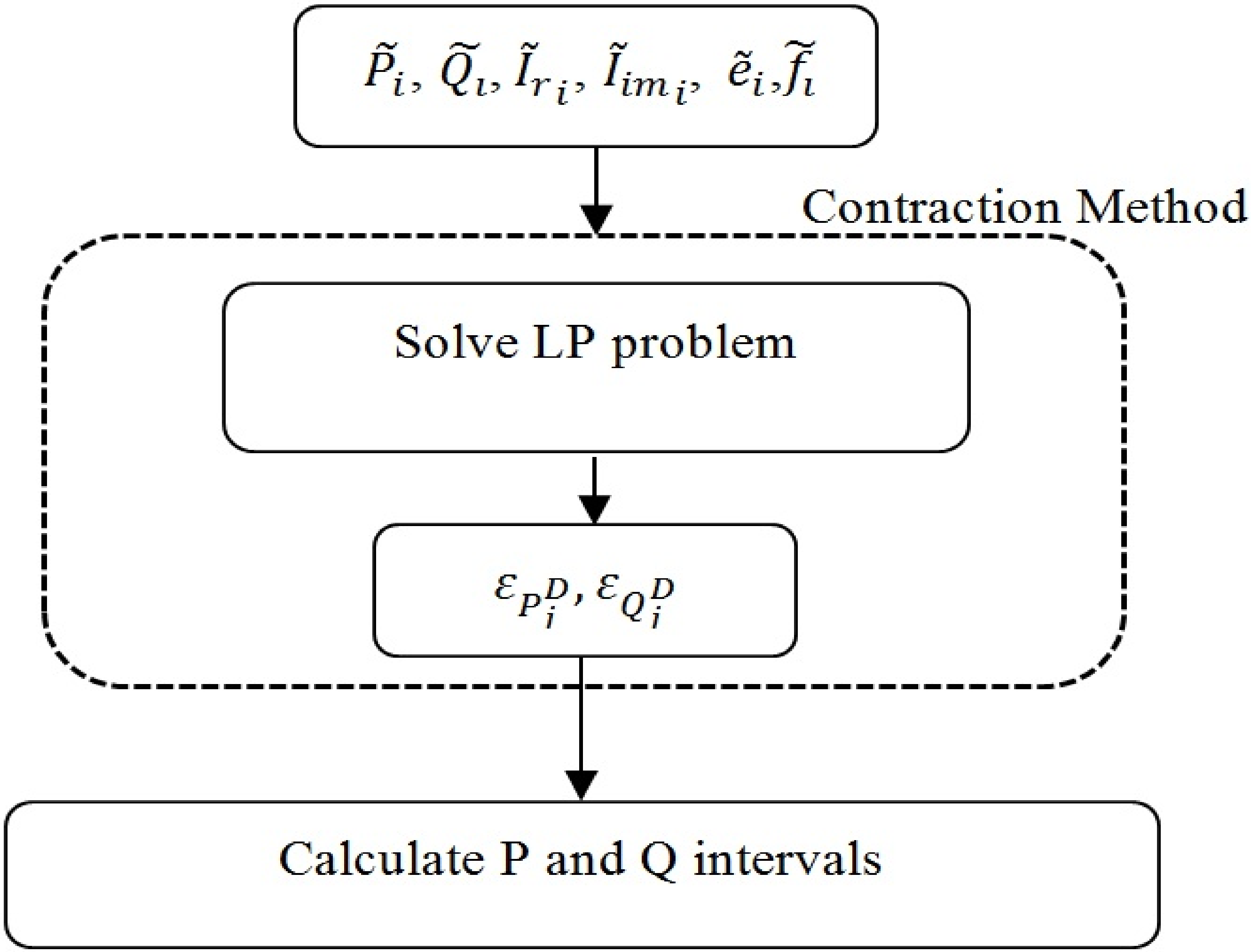
$$P_i^{min} \leq \tilde{P}_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq P_i^{max} \qquad \forall i \in NPG$$

$$Q_i^{min} \leq \tilde{Q}_i\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq Q_i^{max} \qquad \forall i \in NPG$$

$$I_{ij}^{min^2} \leq \tilde{I}_{r_{ij}}^{\,2}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) + \tilde{I}_{im_{ij}}^{\,2}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq I_{ij}^{max^2} \qquad \forall ij \in L$$

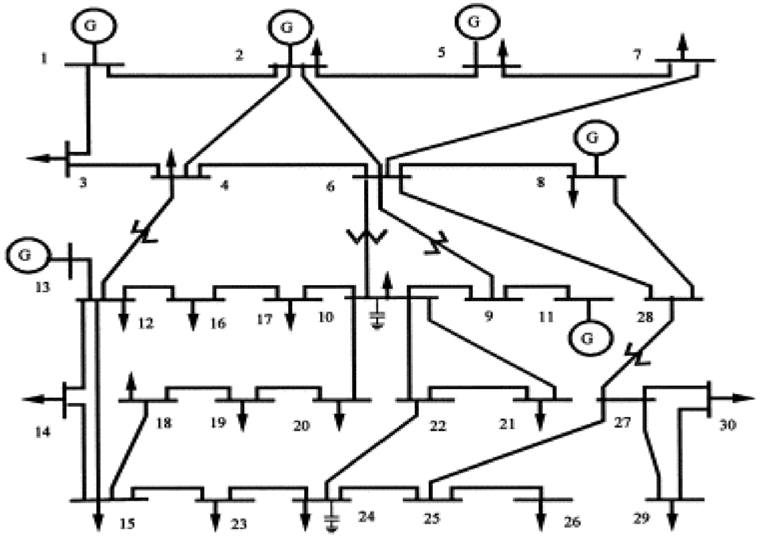
$$V_i^{min^2} \leq \tilde{V}_i^{\,2}\left(\varepsilon_{P_j^D}, \varepsilon_{Q_j^D}\right) \leq V_i^{max^2} \qquad \forall i \in N$$

Contraction Method

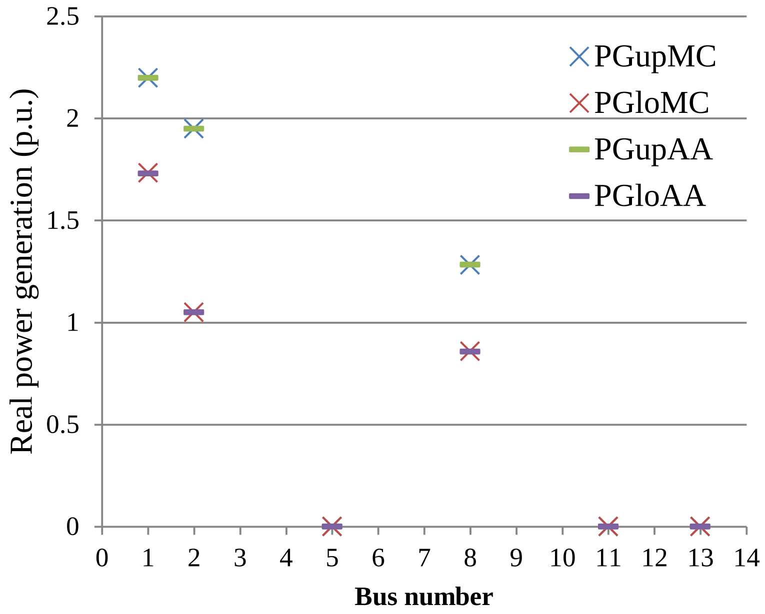


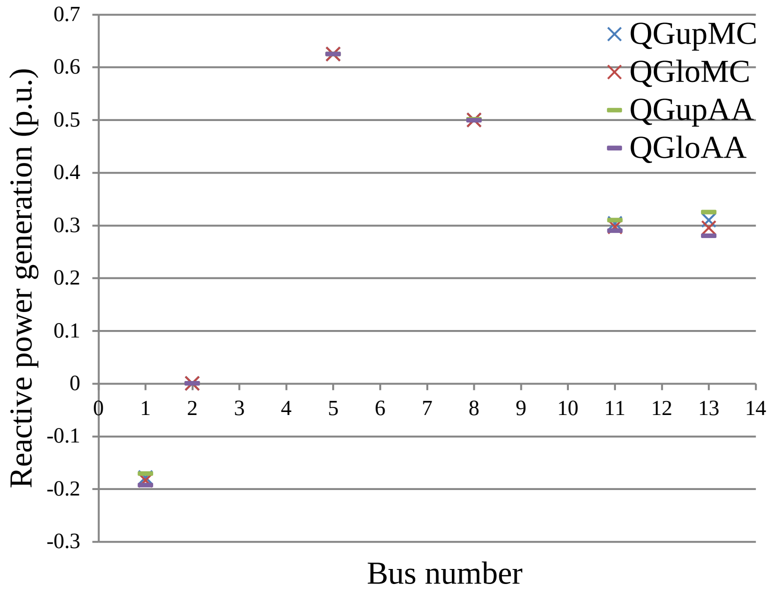
Contraction Method

- AA is implemented in GAMS, for IEEE 30-bus benchmark system and a real 1211-bus European system.
- Results are compared with Monte-Carlo simulation.
- MCS uses:
  - 3000 iterations
  - Uniform distribution

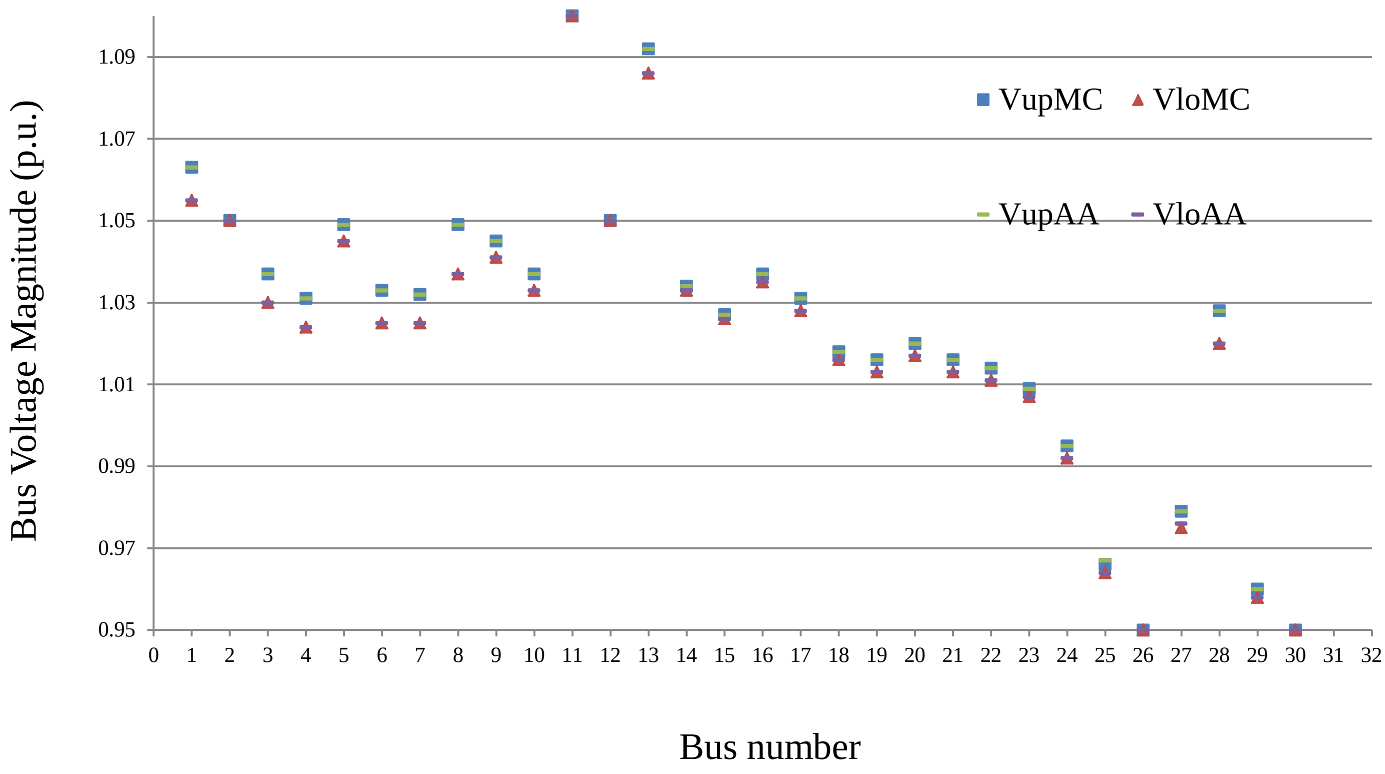


AA v.s. MCS Real and Reactive Power Ranges

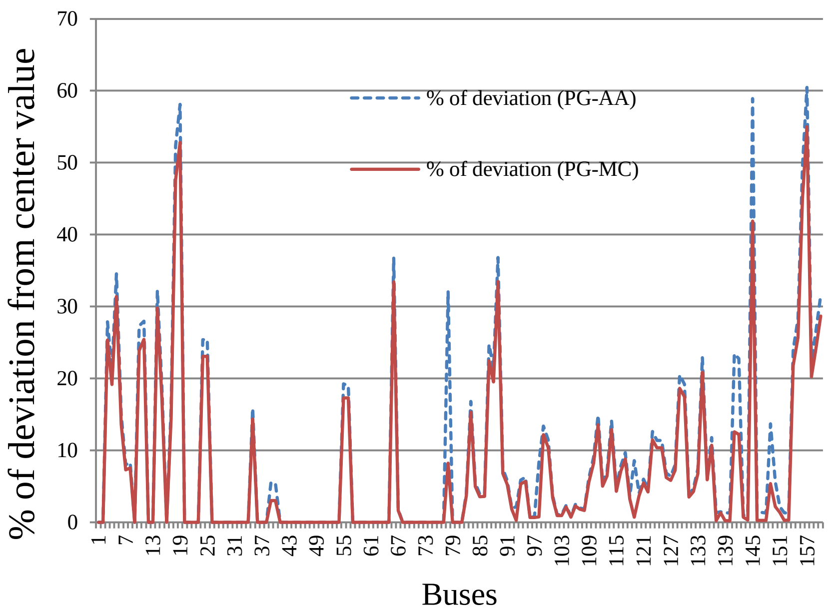


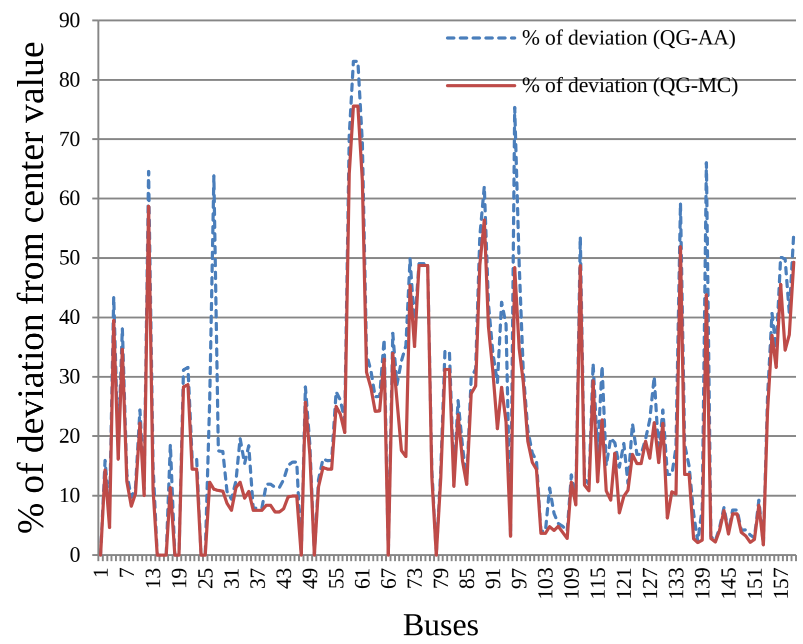


AA v.s. MCS Bus Voltage Magnitude Intervals

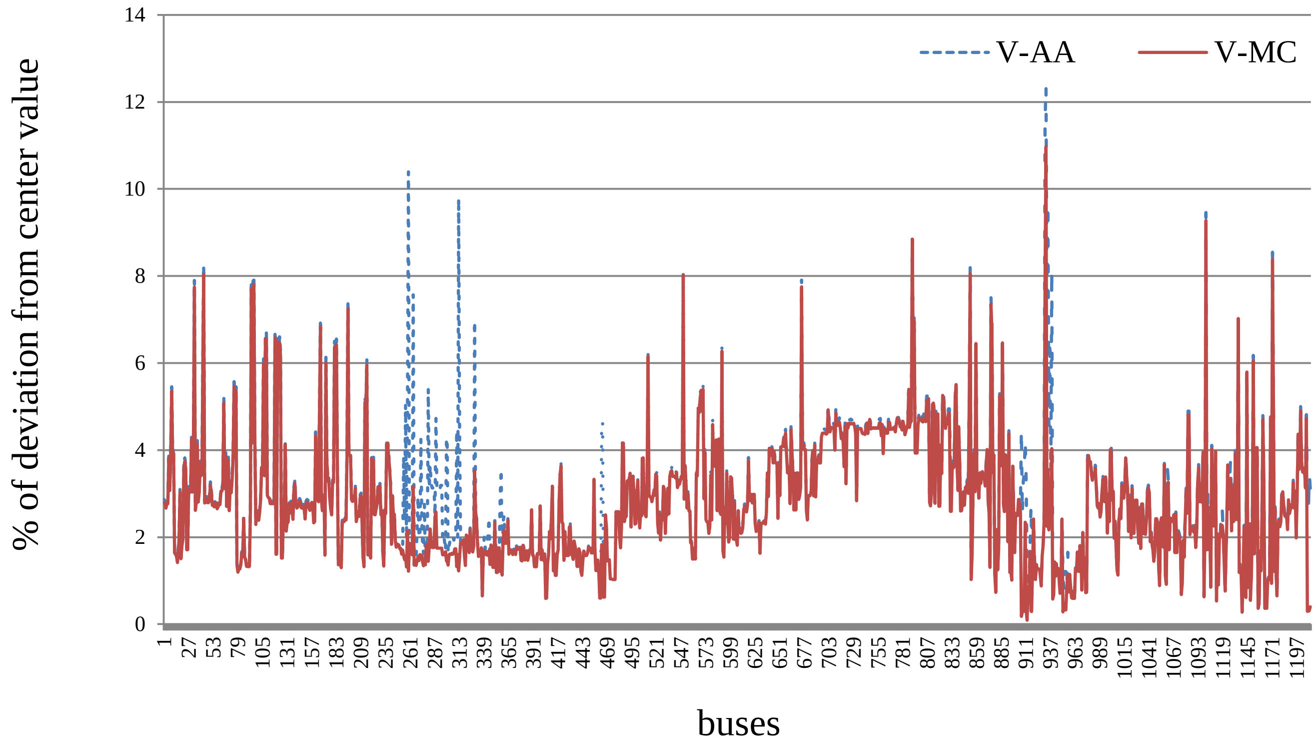


AA v.s. MCS Real and Reactive Power (Thermals)





AA v.s. MCS Bus Voltage Magnitudes



Results: Thermal Generators' output range

Total Thermal Reserve	AA-based Method (GW)	MCS-based Method (GW)	% of Error
Maximum	143.9	142.6	0.91%
Minimum	131.1	128.2	2.21%

Future Work

- Develop a computationally efficient and numerically accurate AA-based model to solve the stochastic UC model with intermittent sources of energy such as wind and solar.

- Provide a detailed comparison of the AA-based solution with the other available solutions such as MCS in regards to its computational efficiency and numerical accuracy.

- Use AA-based method to develop local marginal price intervals, due to uncertainties in the system.

Thanks

Research Papers/ Publications from this work

[1] M. Pirnia, C. Canizares and K. Bhattacharya, “Revisiting the Power Flow Problem Based on a Mixed Complementarity Formulation Approach”, *IET Generation*, *Transmission & Distribution* ,2013.

[2] M. Pirnia, C. Canizares and K. Bhattacharya, A. Vaccaro, "An Affine Arithmetic Method to Solve the Stochastic Power Flow Problem Based on a Mixed Complementarity Formulation” ,2012 IEEE Power & Energy Society Annual General Meeting, San Diego, USA.

Jacobian-Based Sensitivity Analysis

$$f(x,p) = 0$$

$$\frac{\partial f(x_0)}{\partial x} dx + \frac{\partial f(x_0)}{\partial p} dp = 0$$

$$\frac{\Delta x}{\Delta p} \approx \frac{dx}{dp} = -\left[\frac{\partial f(x_0)}{\partial x}\right]^{-1} \frac{\partial f(x_0)}{\partial p}$$

Chebyshev Approximation

- An approximation that minimize the maximum absolute error.

- If f is a bounded and continuous function from some closed and bounded interval $I = [a\,, b]$, then the Chebyshev approximation is $\alpha x + \zeta$

    - $\alpha = \frac{f(b)-f(a)}{b-a} = f'(u)$
    - ζ is such that $\alpha u + \zeta = (f(u) + r(u))/2$

AA-Based PF Formula

Affine Current Calculations:

$$\tilde{I} = \mathrm{Y}\,|\tilde{\mathrm{V}}|$$

$$\tilde{I} = (\mathrm{G} + j\mathrm{B})(\tilde{e} + j\tilde{f})$$

$$\tilde{I}_r = G\tilde{e} - B\tilde{f}$$

$$\tilde{I}_{im} = G\tilde{f} + B\tilde{e}$$

$$\tilde{I}_{r_i} = I_{r_{i,0}} + \sum_{j\epsilon N} I_{r_{i,j}}^{\,P}\,\varepsilon_{P_j} + \sum_{j\epsilon N} I_{r_{i,j}}^{\,Q}\,\varepsilon_{Q_j} \qquad \forall i,j \in N$$

$$\tilde{I}_{im_i} = I_{im,0} + \sum_{j\epsilon N} I_{im_{i,j}}^{\,P}\,\varepsilon_{P_j} + \sum_{j\epsilon N} I_{r_{i,j}}^{\,Q}\,\varepsilon_{Q_j} \qquad \forall i,j \in N$$

Affine Real and Reactive Power calculation:

$$P_{i,0} = e_{i,0}\,I_{r_{i,0}} + f_{i,0}\,I_{im_{i,0}}$$

$$Q_{i,0} = f_{i,0}\,I_{r_{i,0}} - e_{i,0}\,I_{im_{i,0}}$$

$$P_{i,j}^P = e_{i,0}\,I_{r_{i,j}}^{\,P} + I_{r_{i,0}}\,e_{i,j}^P + f_{i,0}\,I_{im_{i,j}}^{\,P} + I_{im_{i,0}}\,f_{i,j}^P$$

$$Q_{i,j}^P = f_{i,0}\,I_{r_{i,j}}^{\,P} + I_{r_{i,0}}\,f_{i,j}^P - e_{i,0}\,I_{im_{i,j}}^{\,P} - I_{im_{i,0}}\,e_{i,j}^P$$

$$P_i^T = \sum_j |e_{i,j}^P| \sum_j |I_{r_{i,j}}^{\,P}| + \sum_j |f_{i,j}^P| \sum_j |I_{im_{i,j}}^{\,P}|$$

$$Q_i^T = \sum_j |f_{i,j}^P| \sum_j |I_{r_{i,j}}^{\,P}| - \sum_j |e_{i,j}^P| \sum_j |I_{im_{i,j}}^{\,P}|$$

$$\tilde{P}_i = P_{i,0} + \sum_j P_{i,j}^P\,\varepsilon_{P_j} + \sum_j P_{i,j}^Q\,\varepsilon_{Q_j} + P_i^T\,\varepsilon_{T_i}$$

$$\tilde{Q}_i = Q_{i,0} + \sum_j Q_{i,j}^P\,\varepsilon_{Q_j} + \sum_j Q_{i,j}^Q\,\varepsilon_{Q_j} + Q_i^T\,\varepsilon_{T_i}$$

Convolution Method

Probabilistic power injection is a function of random variables, generation and load thus the result is a random variable with a PDF

Univariate function

$$y = g(x)$$

$$P(Y \le y) = P[X \le g(y)^{-1}]$$
$$F_Y(y) = F_X(g(y)^{-1})$$
$$= \int_{-\infty}^{g(y)^{-1}} f_x(x)dx$$
$$f_Y(y) = f_X(g(y)^{-1})\frac{dg^{-1}}{dy}$$

Example: $Z = x + y$

$$f_Z(z) = \int_{-\infty}^{\infty} f_{XY}(z-y, y)dy$$
$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y)f_Y(y)dy$$

Bivariate function

$$z = g(x, y)$$
$$x = g^{-1}(z, y) \text{ and } y = g^{-1}(x, z)$$

$$F_Z(z) = P(g(x, y) \le z) = P[X \le g(y)^{-1}]$$

$$F_Z(z) = F_Z(g(x, y) \le z)$$
$$= \iint_{-\infty}^{g^{-1}} f_{X,Y}(x, y)dx\,dy = \iint_{-\infty}^{z} f_{XY}(g^{-1}, y)\left|\frac{dg^{-1}}{dy}\right| dzdy$$
$$f_Z(z) = \int_{-\infty}^{\infty} f_{XY}(g^{-1}, y)\left|\frac{dg^{-1}}{dy}\right| dx$$