Computational Performance of Solution Techniques Applied to the ACOPF

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Staff Technical Conference
Federal Energy Regulatory Commission
Washington DC
June 24-26, 2013

Views expressed are not necessarily those of the Commission.

Overview

- Introduction
- Experimental Design
- Performance Benchmarking
- Numerical Results
- Discussion

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INTRODUCTION

- 1962: Carpentier formulates the ACOPF

  :

- 2013: DCOPF with AC feasibility in the RT energy markets

  :

Integrated DER/DR? Smarter grids? Better market design?

ACOPF advancements over the past 50 years:

- Proposed and published algorithms claim to be …

  faster, converges more robustly, etc.

- Studies published to-date lack rigorous experimental design and benchmarking
- Difficult to compare competing approaches
- Collaborative research initiatives
- What's *our* learning curve here?

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Experimental Design

Sample Set

Solver	Formulation	Initialization	Test Problem	Starting Point
Conopt	PSV	Bθ	118-bus	1 Sample (Bθ, per test problem)
Ipopt	RSV	Uniform	300-bus	100 Samples (Uniform, per test problem)
Knitro	RIV	Hot	2383-bus	100 Samples (Hot, per test problem)
Minos			2736-bus	
Snopt			2746-bus	
			3012-bus	
Solution Technique			3120-bus	

- Solution Technique :=

  (Solver, Formulation, Initialization Method)

- Simulation Record <unique ID> :=

  (Solution Technique, Test Problem, Unique Starting Point)

Experimental Design

Simulation Assumptions

Solver	Formulation	Initialization	Test Problem	Starting Point
Conopt	PSV	Bθ	118-bus	1 Sample (Bθ, per test problem)
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- **Variance Reduction**
  - Common Tolerances on Commercial Solvers
  - Under-utilization system processor and memory resources to circumvent bottlenecks
  - Test the solution techniques with the same set of starting points for the sample of simulations

Experimental Design

Nonlinear Problem Formulation

min Quadratic Cost Function

s.t. Real and Reactive Power Balancing (EQ nonconvex)

Generator Real Power Limits (LB nonconvex, UB convex)

Generator Reactive Power Limits (LB nonconvex, UB convex)

Voltage Magnitude Limits (LB nonconvex, UB convex)

Not Included…

Thermal Line Limits

Generator Capability Curves

Maximum Phase Angle Difference

Discrete Controls

Contingencies

… And Yet Still Difficult!

Experimental Design

Power Balancing Constraints

Polar Power-Voltage Power Flow Formulation (PSV)

$$v_n \sum_{m \in N} v_m \left(g_{nm} \cos\theta_{nm} + b_{nm} \sin\theta_{nm} \right) - p_n + p_n^d = 0, \forall n \in N$$

$$v_n \sum_{m \in N} v_m \left(g_{nm} \sin\theta_{nm} - b_{nm} \cos\theta_{nm} \right) - q_n + q_n^d = 0, \forall n \in N$$

Rectangular Power-Voltage Power Flow Formulation (RSV)

$$v_n^r \sum_{m \in N} \left(g_{nm} v_m^r - b_{nm} v_m^j \right) + v_n^j \sum_{m \in N} \left(g_{nm} v_m^j + b_{nm} v_m^r \right) - p_n + p_n^d = 0, \forall n \in N$$

$$v_n^j \sum_{m \in N} \left(g_{nm} v_m^r - b_{nm} v_m^j \right) - v_n^r \sum_{m \in N} \left(g_{nm} v_m^j + b_{nm} v_m^r \right) - q_n + q_n^d = 0, \forall n \in N$$

Rectangular Current Injection Formulation (RIV)

$$i_n^r = \sum_{m \in N} \left(g_{nm} v_m^r - b_{nm} v_m^j \right) \qquad i_n^j = \sum_{m \in N} \left(g_{nm} v_m^j + b_{nm} v_m^r \right), \forall n \in N$$

$$p_n = v_n^r i_n^r + v_n^j i_n^j + p_n^d \qquad q_n = v_n^j i_n^r - v_n^r i_n^j + q_n^d, \forall n \in N$$

Experimental Design

Initialization Methodology

Bθ Start (1 Starting Point per Test Problem)

DCOPF (lossless, convex LP) assuming unitary voltage magnitude [i.e. 1 p.u.] where $\sin(\Theta^{diff}) \approx \Theta^{diff}$ and negligible resistance [i.e. G = 0]

Hot Start (100 Starting Points per Test Problem)

Converged solution of a perturbation to the original problem (equivalent to sequential multi-period dispatch) by shifting the real power load ±10% where

$$P^d = U(0.9P^d, 1.1P^d)$$

Uniform Start (100 Starting Points per Test Problem)

Determine real and reactive power injections through the power balancing equations, where

$$V^{init} = U(V^{min}, V^{max}) \text{ and } \Theta^{diff} = 0$$

Experimental Design

Commercial Solvers

Commercial Solver	Version	Algorithm Method
Conopt	3	Generalized Reduced Gradient
Ipopt	3.10	Interior Point
Knitro	8.0	Interior Point, Active Set
Minos	5.51	Augmented Lagrangian
Snopt	7	Sequential Quadratic Programming

Test Problems

Network	Br	Gen	L	N-Squared (%)	System Cost ($)
118-bus	186	54	99	0.04	129,661
300-bus	411	69	199	0.23	719,725
2383-bus	2896	323	1822	14.33	1,922,928
2736-bus	3269	206	2011	18.90	1,307,832
2746-bus	3279	342	1993	19.03	1,505,109
3012-bus	3572	292	2260	22.90	2,584,227
3120-bus	3693	229	2277	24.57	2,232,988

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Performance Benchmarking

Performance Ratios

We define the following performance ratio for each simulation record

$$\tau_{sfip}^{n} = \frac{t_{sfip}^{n}}{t_{p}^{*}}$$

τ-Ratio

where

t_p^* := Minimum recorded CPU time (seconds)

t := Recorded CPU time (seconds)

s := solver

i := initialization method

p := test problem

n := starting point (n = n(i,p))

Example. Fastest simulation on a test problem is where $\tau_{sfip}^{n} = 1$ which sets t_{p}^{*} —we define this simulation as the ***reference solution***.

Performance Benchmarking

Dolan-Moré Performance Profiles

The performance profile for a given solution technique is defined as the distribution function P_s: R → [0,1] for

$$P_s(\tau) = \frac{\sum_p \omega_p \left| n \in N : \tau_{sfip}^n \le \tau, p \in P \right|}{\sum_p \left| n \in N : p \in P \right|}$$

where

τ := τ-Ratio ([simulation CPU time]/[Best simulation CPU time] for a given test problem)

ω := N-squared (nodes^2) weight

s := solver

i := initialization method

p := test problem

n := starting point (n = n(i,p))

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Numerical Results

Test Bed

- GAMS 23.6.2 Platform
- Intel Zeon E7458 2.4GHz, 64 GB RAM, 64-bit Windows Server 2008 Enterprise
- N-squared Weights (nodes^2)
- Each simulation instance was solved sequentially with a maximum time limit of 20 minutes.
- CPU time for nonconverged instances > 20 minutes

Solver	Formulation	Initialization	Test Problem	Starting Point
Conopt	PSV	Bθ	118-bus	1 Sample (Bθ, per test problem)
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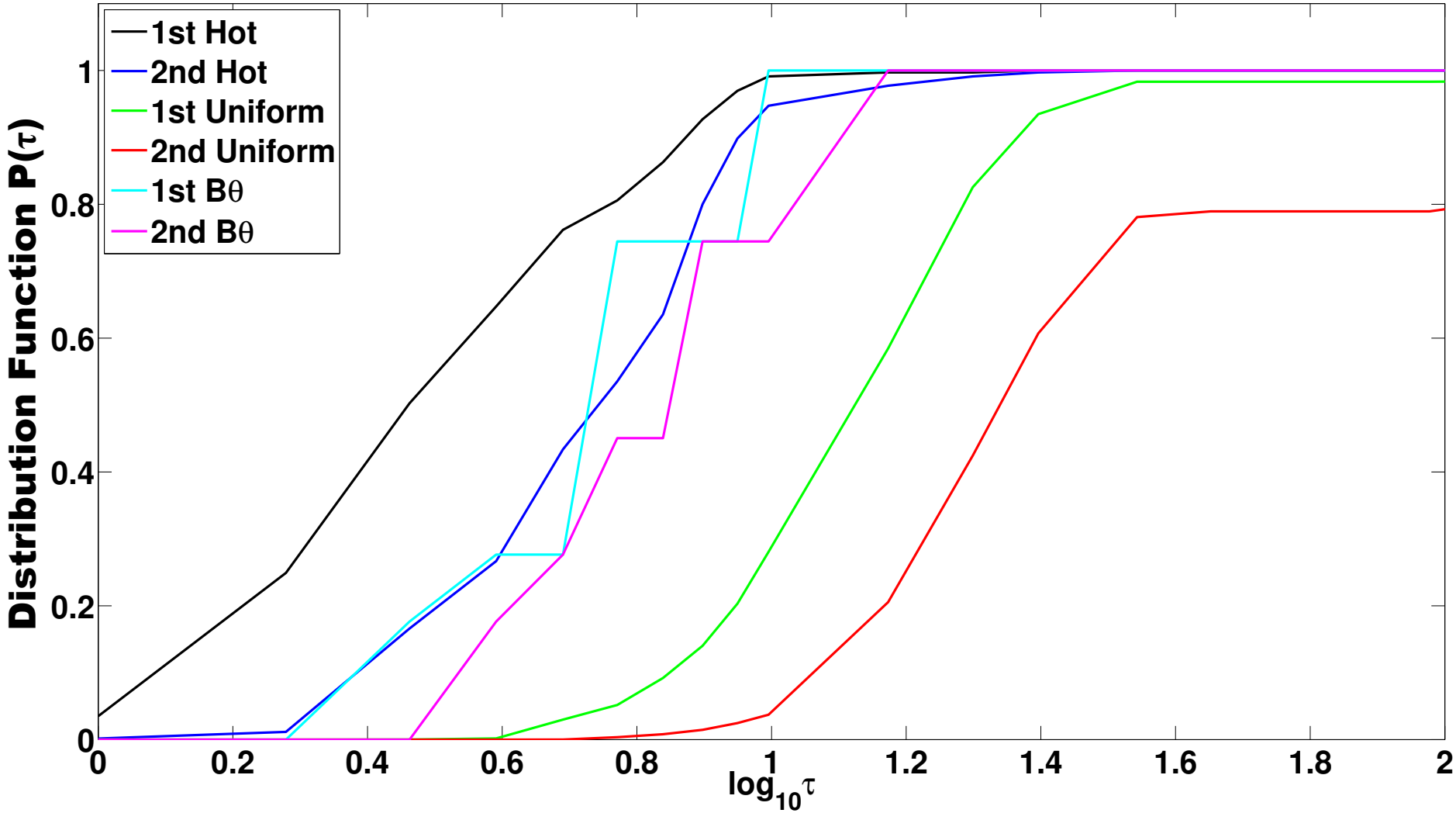
Knitro	RIV	Hot	2383-bus	100 Samples (Hot, per test problem)
Minos			2736-bus	
Snopt			2746-bus	201 Starting Points x 7 Test Problems x 5 Solvers x 3 Formulations = 21,105 Simulation Records
			3012-bus	
Solution Technique			3120-bus	

Numerical Results

Reference Solution

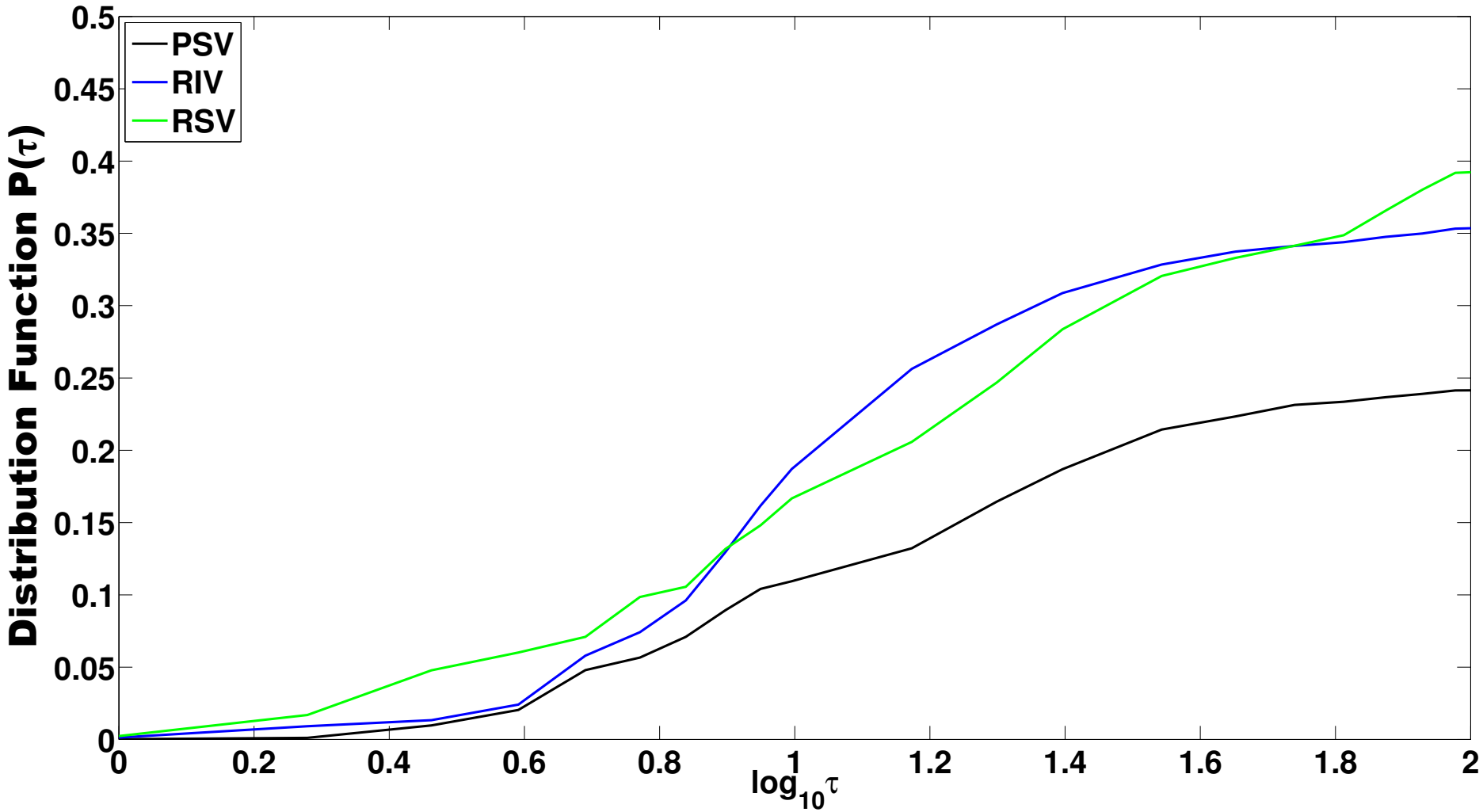
Test Problem	t*p, Minimum CPU Time (s)	Solver	Formulation	Initialization
118-bus	0.1	Conopt	RSV	Hot
300-bus	0.3	Conopt	PSV	Hot
2383-bus	10.3	Minos	RIV	Hot
2736-bus	6.5	Minos	RIV	Hot
2746-bus	11.3	Conopt	RSV	Hot
3012-bus	8.4	Knitro	RIV	Hot
3120-bus	10.1	Conopt	RSV	Hot

Reference solution indicates the simulation record with the minimum CPU time attained to solve the test problem to optimality.



Lowest CPU Time by Starting Point

Hot starts drastically outperform uniform starts.

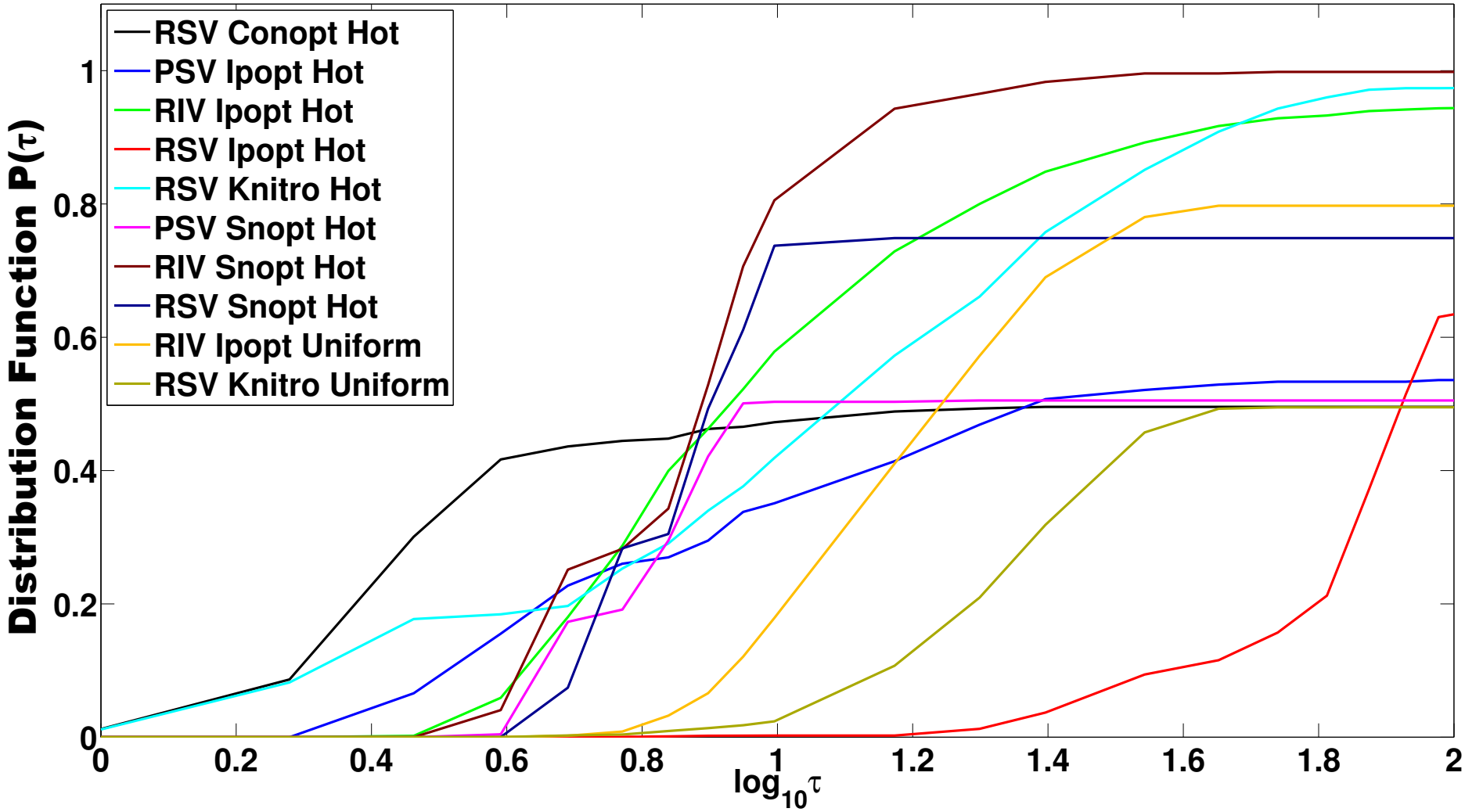


Formulation Performance Profiles

PSV: Polar Power-Voltage Power Flow Formulation

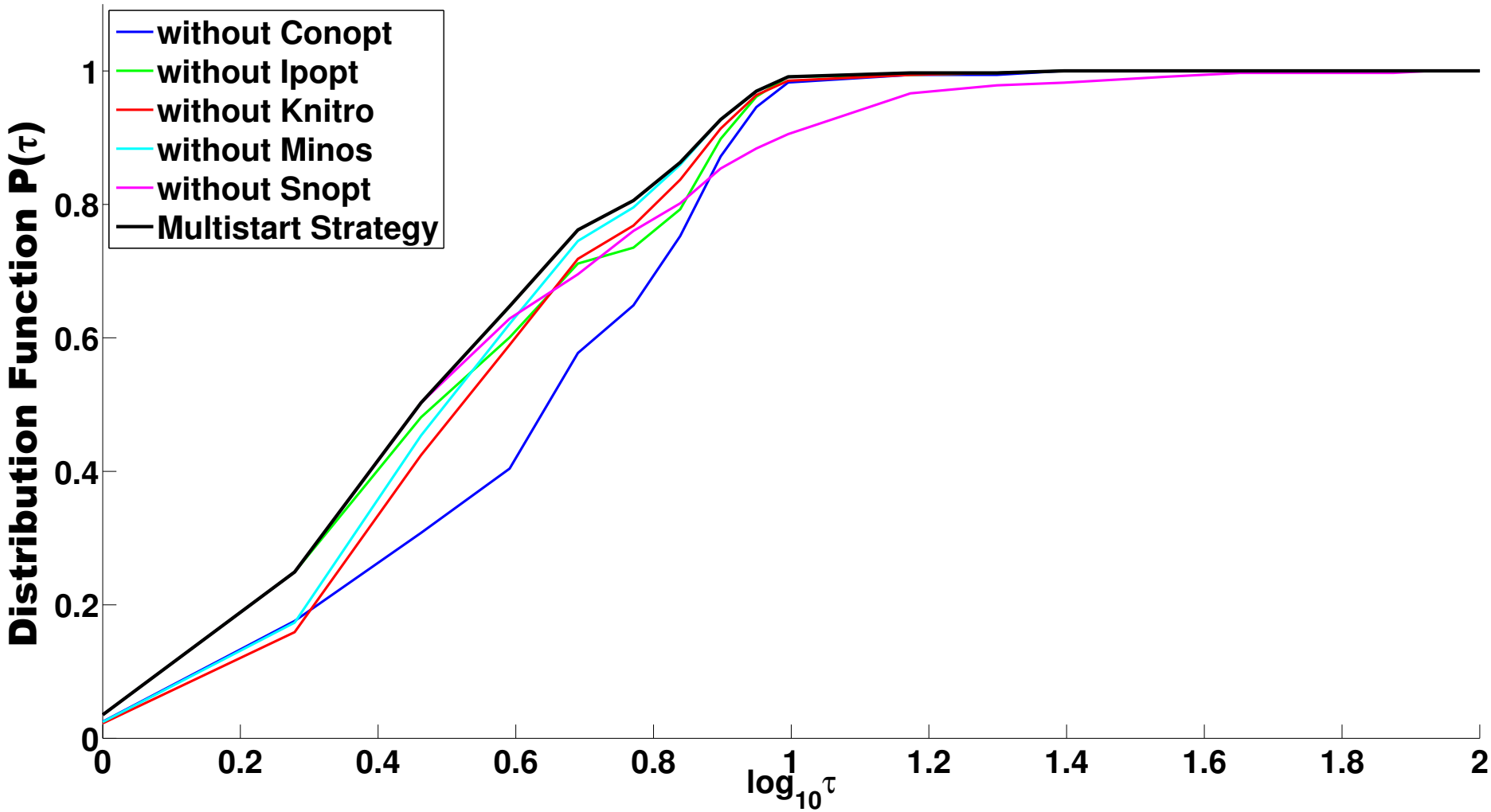
RIV: Rectangular Current Injection Formulation

RSV: Rectangular Power-Voltage Power Flow Formulation



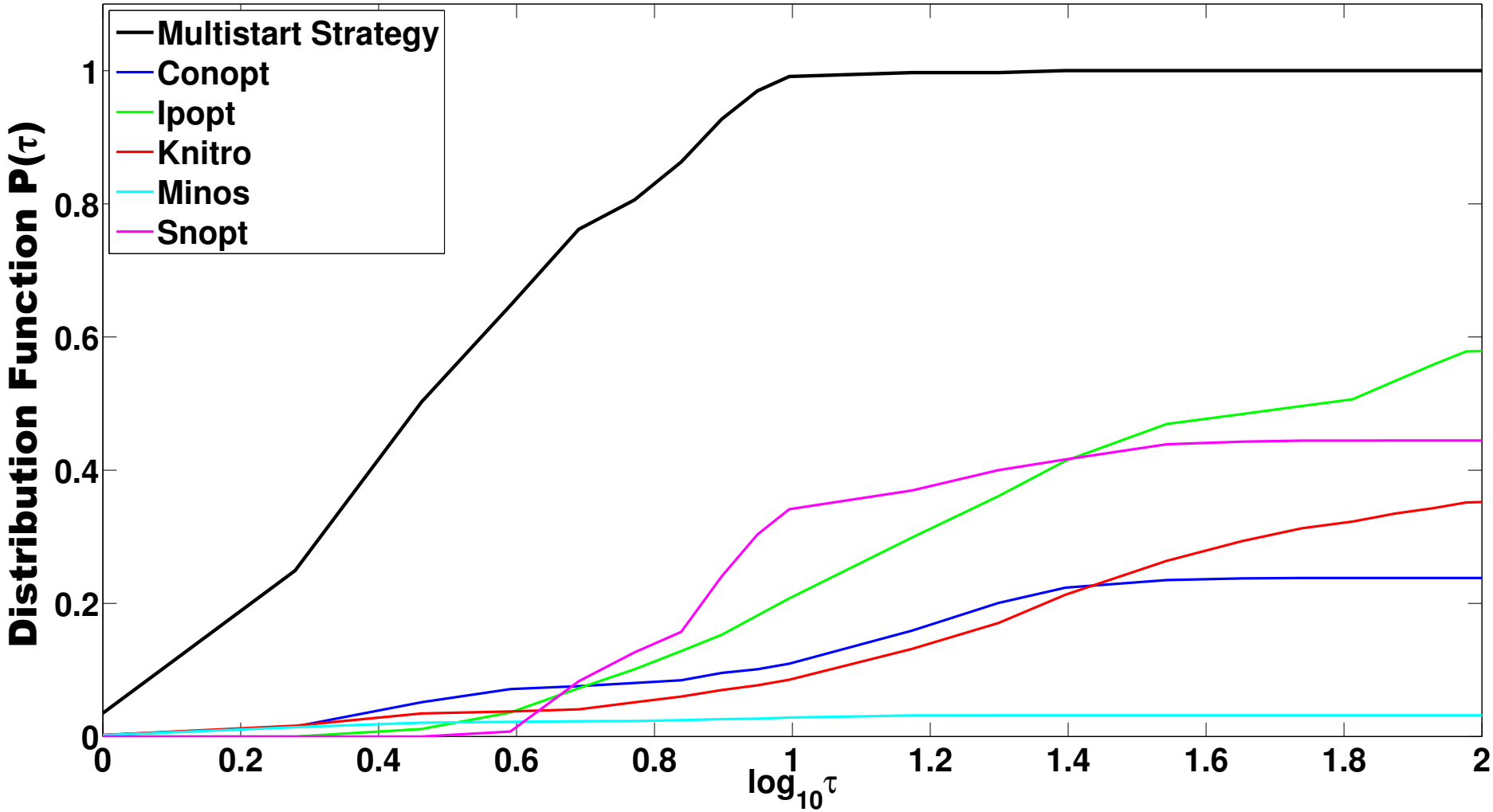
Solution Technique Performance Profiles

Rectangular Current Injection (RIV), Snopt, Hot Initialization: 100% Convergence on 7 Test Problems x 100 Starting Points = 700 unique simulation instances



Impact of Solver on Parallel Processing

Gain intuition regarding the marginal impact of each solver. The remaining performance profiles demonstrate the overall performance when a given solver is excluded from the parallel processing.



Solver Performance Profiles

A comparison of the multistart strategy to each solver as independent solution strategies.

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- **Preliminary Results (i.e. without full AC and SC constraint set)**
  - Clear advantage in a multistart strategy (cloud/grid computing)
  - Rectangular coordinates outperform polar coordinates
  - Uniform initializations indicate a lack of robustness of current solvers applied to the ACOPF

- **Future ACOPF Algorithm Development & Benchmarking**
  - Improved Learning Curve
  - Robust, Replicable, & Reliable
- **Standard Suite of Test Problems**
  - Publicly Available Networks, Intermittent Data, Cost Curves, etc.
  - Includes Thermal Limits
  - Comparable Results
- **Commercial Solvers v. 'In-House'**
  - Customized Parameterization

Thank You!

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For the entire report
and other papers on ACOPF:

http://www.ferc.gov/industries/electric/indus-act/market-planning/opf-papers.asp