Multi-Stage Robust Unit Commitment Considering Wind and Demand Response Uncertainties

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Unified Stochastic and Robust Unit Commitment

$$\begin{aligned}
\min_{y,u,v} \quad & (\mathbf{a}^T u + \mathbf{b}^T v) + \alpha \frac{1}{N} \sum_{n=1}^{N} \mathbf{e}^T \phi(\xi^n) + (1-\alpha) \max_{d \in \mathcal{D}} \min_{x,\phi} \mathbf{e}^T \phi \\
s.t. \quad & \mathbf{A} y + \mathbf{B} u + \mathbf{C} v \geq \mathbf{r}, \\
& \mathbf{F} y - \mathbf{D} q(\xi^n) \leq \mathbf{g}, \quad n = 1, \cdots, N \\
& \mathbf{K} y - \mathbf{P} q(\xi^n) - \mathbf{J} \phi(\xi^n) \leq 0, \quad n = 1, \cdots, N \\
& \mathbf{T} q(\xi^n) \geq \mathbf{S} d(\xi^n) + \mathbf{s}, \quad n = 1, \cdots, N \\
& \mathbf{F} y - \mathbf{D} x(d) \leq \mathbf{g}, \\
& \mathbf{K} y - \mathbf{P} x(d) - \mathbf{J} \phi(d) \leq 0, \\
& \mathbf{T} x(d) \geq \mathbf{S} d + \mathbf{s}, \\
& y, u, v \in \{0, 1\}, x(d), q(\xi^n) \geq 0, \phi(d), \phi(\xi^n) \text{ free}, \forall n
\end{aligned}$$

where

$$\mathcal{D} = \left\{ d \in \mathcal{R}^{|B| \times |T|} : \mathbf{d}^- \leq d \leq \mathbf{d}^+, \mathbf{U}^T d \leq \mathbf{z} \right\}.$$

Ref: Zhao and Guan, Unified Stochastic and Robust Unit Commitment, IEEE Transaction on Power Systems, 2013.

Agenda

- Introduction and Motivation

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- Mathematical Formulation

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- Solution Methodologies

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- Computational Results
- Conclusion

Wind Energy

- Wind power is hard to predict and intermittent
  - Security: system reliability
  - Operation: generation scheduling and curtailment
- Consider renewable energy a higher priority over the other conventional generation sources.
- Physical constraints of other non-wind units, such as ramping rates and minimum up/down times, are influenced.
- Challenges on how to commit thermal generation units to accommodate intermittent wind power.

UC with Wind

- Significant research progress made recently on UC with wind.
- Simulate wind realizations: Stochastic unit commitment models, e.g., [4] and lots of others....
- Ensure high utilization:
  - A large portion of wind power can be utilized with a high probability: chance constrained model [6].
  - Wind power within an interval defined by quantiles is ensured to be utilized: robust unit commitment model, e.g., [13], [7], and several others....

Demand Response

- Demand Response Program (DRP) encourages the demand side to voluntarily schedule the consumption based on the price signal.
  - For load-serve entities
    - Reduce peak-hour power supply and lower the capacity requirement.
  - For customers
    - Adjust electricity demand from high price periods to low price periods to reduce electricity usage cost.
  - For Independent System Operators (ISO)
    - Accommodate wind power uncertainty, balance the electricity consumption and production...

Demand Response Curve

- DR was mostly modeled as a "fixed" demand curve
- Price-elastic demand curve
  - Characterized by price elasticity - the sensitivity of electricity demand (Q) with respect to the change of price (P), i.e., $\alpha = \frac{\Delta Q/Q}{\Delta P/P}$
  - Concept of "inelastic demand" and "elastic demand"
    - Inelastic demand: critical loads like hospitals and airports
    - Elastic demand: depend on time-varying price
  - DR vs auxiliary services market

Demand Response Curve

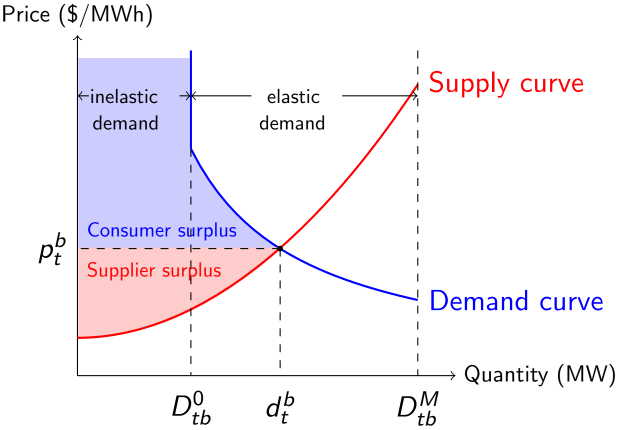

Figure: An Example of Price-elastic Demand Curve

Our Contributions

- Both wind power output and demand response uncertainties in the Reliability Unit Commitment Run (or Reliability Assessment Commitment Run).
- Multi-stage robust optimization model to address both uncertainties, instead of previous two-stage robust optimization approaches.
- A tractable solution approach to solve the multi-stage robust optimization problem.

Mathematical Formulation - Notation

Parameters

SU_i^b/SD_i^b	Start-up/shut-down cost for generator i at bus b
MU_i^b/MD_i^b	Minimum-up/Minimum-down time for generator i at bus b
MA_i^b/MI_i^b	Maximal/Minimal generation capacity of generator i at bus b
RU_i^b/RD_i^b	Ramp-up/Ramp-down limit for generator i at bus b
D_{tb}^0/D_{tb}^M	The inelastic/maximum demand at bus b in time period t.
U_{ij}	Maximal capacity for transmission line from bus i to bus j
F_{ij}^b	Distribution factor for transmission line from bus i to bus j due to the net injection at bus b

Mathematical Formulation - Notation

Decision variables

y_{it}^b	Generator i is on $(= 1)$ or not $(= 0)$ in period t at bus b
u_{it}^b	Generator i is started up $(= 1)$ or not $(= 0)$ in period t at bus b
v_{it}^b	Generator i is shut down $(= 1)$ or not $(= 0)$ in period t at bus b
x_{it}^b	Amount of electricity generated by generator i in period t at bus b
d_t^b	Electricity demand in period t at bus b

Nominal Model

- Objective:
  $\max \sum_{b=1}^{B} \sum_{t=1}^{T} r_t^b(d_t^b) - \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{i=1}^{G_b} (f_i^b(x_{it}^b) + SU_i^b u_{it}^b + SD_i^b v_{it}^b)$
- Constraints: Minimum-up/down time constraints (MM)

$$-y_{i(t-1)}^b + y_{it}^b - y_{ik}^b \leq 0,$$
$$(1 \leq k - (t-1) \leq MU_i^b,$$
$$t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$
$$y_{i(t-1)}^b - y_{it}^b + y_{ik}^b \leq 1,$$
$$(1 \leq k - (t-1) \leq MD_i^b,$$
$$t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$

Start-up/Shut-down constraints (SS)

$$-y_{i(t-1)}^b + y_{it}^b - u_{it}^b \leq 0,$$
$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$
$$y_{i(t-1)}^b - y_{it}^b - v_{it}^b \leq 0,$$
$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$

Nominal Model

- Constraints: Ramp-up/down rate constraints (RR)

$$x_{it}^b - x_{i(t-1)}^b \leq y_{i(t-1)}^b RU_i^b + (1 - y_{i(t-1)}^b) MA_i^b,$$
$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$
$$x_{i(t-1)}^b - x_{it}^b \leq y_{it}^b RD_i^b + (1 - y_{it}^b) MA_i^b,$$
$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$

Generation capacity constraints (GC)

$$MI_i^b y_{it}^b \leq x_{it}^b \leq MA_i^b y_{it}^b,$$
$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$

Load balance (SD)

$$\sum_{b=1}^{B} (\sum_{i=1}^{G_b} x_{it}^b + w_t^b) = \sum_{b=1}^{B} d_t^b,$$
$$(t = 1, 2, \cdots, T)$$

DC transmission constraint (TC)

$$-U_{ij} \leq \sum_{b=1}^{B} F_{ij}^b (\sum_{l=1}^{G_b} x_{lt}^b + w_t^b - d_t^b) \leq U_{ij}.$$
$$((i,j) \in \Omega, t = 1, 2, \cdots, T)$$

Load bound (LB)

$$D_{tb}^0 \leq d_t^b \leq D_{tb}^M. \ (t = 1, 2, \cdots, T, b = 1, 2, \cdots, B)$$

Multi-Stage Robust Optimization Model

- Linearize the nonlinear terms of the objective function

$$
\begin{aligned}
z^{\mathrm{R}} = \max & \sum_{b=1}^{B}\sum_{t=1}^{T} r_t^b(d_t^b) - \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}(f_i^b(x_{it}^b) + SU_i^b u_{it}^b + SD_i^b v_{it}^b) \\
\text{s.t.} \quad & \text{(SS),(MM),(RR),(GC),(SD),(TC),(LB),} \\
& y_{it}^b, u_{it}^b, v_{it}^b \in \{0,1\}, \text{ and } y_{i1}^b = 0. \\
& (t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)
\end{aligned}
$$

Multi-Stage Robust Optimization Model

- Linearize the nonlinear terms of the objective function
  - $r_t^b(d_t^b)$

$$z^{\mathrm{R}} = \max \sum_{b=1}^{B} \sum_{t=1}^{T} r_t^b(d_t^b) - \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{i=1}^{G_b} (f_i^b(x_{it}^b) + SU_i^b u_{it}^b + SD_i^b v_{it}^b)$$

$$\text{s.t.} \quad \text{(SS),(MM),(RR),(GC),(SD),(TC),}$$

$$y_{it}^b, u_{it}^b, v_{it}^b \in \{0,1\}, \text{ and } y_{i1}^b = 0,$$

$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$

Multi-Stage Robust Optimization Model

- Linearize the nonlinear terms of the objective function
  - $r_t^b(d_t^b)$
  - $f_i^b(x_{it}^b)$

$$z^{\mathrm{R}} = \max \sum_{b=1}^{B}\sum_{t=1}^{T} r_t^b(d_t^b) - \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}\left(f_i^b(x_{it}^b) + SU_i^b u_{it}^b + SD_i^b v_{it}^b\right)$$

$$\text{s.t.} \quad \text{(SS),(MM),(RR),(GC),(SD),(TC),}$$

$$y_{it}^b, u_{it}^b, v_{it}^b \in \{0,1\}, \text{ and } y_{i1}^b = 0,$$

$$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$$

Linearize $r_t^b(d_t^b)$

- If the elasticity value is constant for the entire curve, the price-elastic demand curve will be: $d_t^b = A_t^b (p_t^b)^{\alpha_t^b}$.
- Linearize the price-elastic demand curve by a step-wise function.

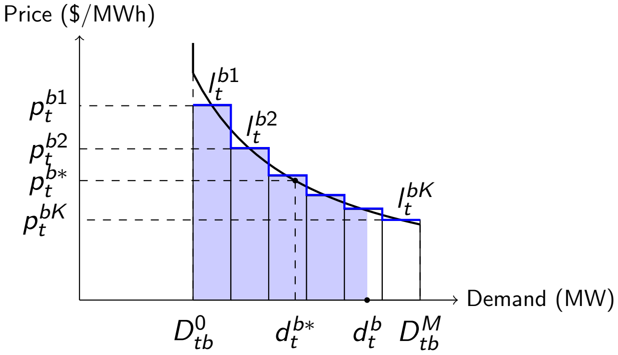

Figure: Approximation of the Price-elastic Demand Curve

Linearize $r_t^b(d_t^b)$

- Linearize the curve by a K-piece step-wise function.
  - $r_t^b(d_t^b) = \sum_{k=1}^{K} p_t^{bk} h_t^{bk}, d_t^b = \sum_{k=1}^{K} h_t^{bk}, 0 \le h_t^k \le \ell_t^{bk}$
    $(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b, k = 1, 2, \cdots, K)$

$$z^{\mathrm{R}} = \max \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{k=1}^{K} p_t^{bk} h_t^{bk} - \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{i=1}^{G_b} (f_t^b(x_{it}^b) + SU_i^b u_{it}^b + SD_i^b v_{it}^b)$$

s.t. (SS),(MM),(RR),(GC),(SD),(TC),(LB)

$d_t^b = \sum_{k=1}^{K} h_t^{bk}$,

$0 \le h_t^k \le \ell_t^{bk}$,

$y_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}$, and $y_{i1}^b = 0$.

$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$

Linearize $f_i^b(x_{it}^b)$

- Linearize the cost function by an N-piece piecewise linear function:
  - $\phi_{it}^b \geq \beta_{it}^{bj} y_{it}^b + \gamma_{it}^{bj} x_{it}^b$.
    $(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b, j = 1, 2, \cdots, N)$

$$z^{\mathrm{R}} = \max \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{k=1}^{K} p_t^{bk} h_t^{bk} - \sum_{b=1}^{B} \sum_{t=1}^{T} \sum_{i=1}^{G_b} (\phi_{it}^b + SU_i^b u_{it}^b + SD_i^b v_{it}^b)$$

s.t. (SS),(MM),(RR),(GC),(SD),(TC),(LB),

$d_t^b = \sum_{k=1}^{K} h_t^{bk}, 0 \leq h_t^k \leq l_t^{bk}$,

$\phi_{it}^b \geq \beta_{it}^{bj} y_{it}^b + \gamma_{it}^{bj} x_{it}^b$,

$y_{it}^b, u_{it}^b, v_{it}^b \in \{0, 1\}$, and $y_{i1}^b = 0$.

$(t = 1, 2, \cdots, T, b = 1, 2, \cdots, B, i = 1, 2, \cdots, G_b)$

Uncertainty Sets

Wind power output formulation

- $$\mathcal{W} := \left\{ w \in \mathrm{R}^{|B|\times|T|} : w_t^b = W_t^{b*} + z_t^{b+} W_t^{b+} - z_t^{b-} W_t^{b-}, \right.$$
$$\left. \sum_{t=1}^{T} (z_t^{b+} + z_t^{b-}) \leq \pi^b, \forall t, \forall b \right\}$$
- Remark:
  - W_t^{b*} is the predicted value for wind power output in period t at bus b, and W_t^{b+}, W_t^{b-} are the allowed maximum deviations above and below W_t^{b*}, respectively. This interval can usually be generated by using quantiles.
  - π^b is cardinality budget to restrict the number of time periods in which the wind power output is far away from its forecasted value at bus b.
  - Wind power reaches upper bound ($z_t^{b+} = 1$), lower bound ($z_t^{b-} = 1$), or forecasted value ($z_t^{b+} = z_t^{b-} = 0$).

Uncertainty Sets

Uncertain demand response curve formulation

- $\Pi = \{\epsilon : -\hat{\epsilon}_t^{bk} \leq \epsilon_t^{bk} \leq \hat{\epsilon}_t^{bk}, -\kappa_t^b \leq \sum_{k \in K} \epsilon_t^{bk} \leq \kappa_t^b, \forall t = 1, \cdots, T, \forall k = 1, \cdots, K, \forall b = 1, \cdots, B.\}$

- Remark:

  ϵ_t^{bk} is the deviation of p_t^{bk},

  $\hat{\epsilon}_t^{bk}$ is the upper limit of ϵ_t^{bk},

  κ_t^b is used to adjust the robustness.

Uncertainty Sets

Uncertain demand response curve representation:

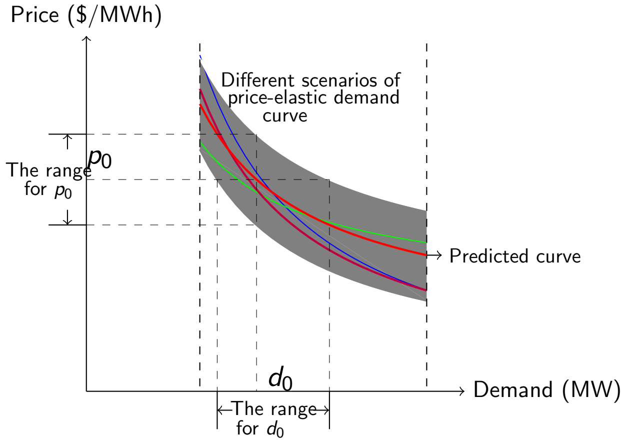

Figure: The Uncertainty of Price-elastic Demand Curve

Multi-Stage Robust Optimization Formulation

- The formulation of the multi-stage robust optimization problem is:

$$\max_{y,u,v} -\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}(SU_i^b u_{it}^b + SD_i^b v_{it}^b) + \min_{w\in\mathcal{W}} \max_{x,h\in\mathcal{X}} \Big(\min_{\epsilon\in\Pi} \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{k=1}^{K}(p_t^{bk*} + \epsilon_t^{bk})h_t^{bk} - \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}\phi_{it}^b \Big)$$

$$\begin{aligned}
\text{s.t.}\quad & (\text{SS}),(\text{MM}),\\
& y_{it}^b, u_{it}^b, v_{it}^b \in \{0,1\}, \text{ and } y_{i1}^b = 0,\\
& (t = 1,2,\cdots,T, b = 1,2,\cdots,B, i = 1,2,\cdots,G_b)\\
& \mathcal{X} = \Big\{(x,h) : (RR),(GC),(SD),(TC),(LB)\\
& \qquad \phi_{it}^b \geq \alpha_{it}^{bj} y_{it}^b + \beta_{it}^{bj} x_{it}^b,\\
& \qquad (t = 1,2,\cdots,T, b = 1,2,\cdots,B, i = 1,2,\cdots,G_b)\\
& \qquad d_t^b = \textstyle\sum_{k=1}^{K} h_t^{bk},\ 0 \leq h_t^{bk} \leq l_t^{bk}.\\
& \qquad (t = 1,2,\cdots,T, b = 1,2,\cdots,B, k = 1,2,\ldots,K)\Big\}
\end{aligned}$$

Multi-Stage Robust Optimization Formulation

- The formulation of the multi-stage robust optimization problem is:
  - First stage: unit commitment decisions y, u, v

$$\max_{y,u,v} -\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}(SU_i^b u_{it}^b + SD_i^b v_{it}^b) + \min_{w\in\mathcal{W}} \max_{x,h\in\mathcal{X}} \left(\min_{\epsilon\in\Pi} \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{k=1}^{K}(p_t^{bk*} + \epsilon_t^{bk})h_t^{bk} - \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}\phi_{it}^b \right)$$

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Multi-Stage Robust Optimization Formulation

- The formulation of the multi-stage robust optimization problem is:
  - First stage: unit commitment decisions y, u, v
  - Second stage: economic dispatch amount under the worst-case wind power output scenario

$$\max_{y,u,v} -\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}(SU_i^b u_{it}^b + SD_i^b v_{it}^b) + \min_{w\in\mathcal{W}} \max_{x,h\in\mathcal{X}} \left(\min_{\epsilon\in\Pi} \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{k=1}^{K}(p_t^{bk*} + \epsilon_t^{bk})h_t^{bk} - \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}\phi_{it}^b \right)$$

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& \qquad (t = 1,2,\cdots,T, b = 1,2,\cdots,B, k = 1,2,\ldots,K)\Big\}
\end{aligned}$$

Multi-Stage Robust Optimization Formulation

- The formulation of the multi-stage robust optimization problem is:
  - First stage: unit commitment decisions y, u, v
  - Second stage: economic dispatch amount under the worst-case wind power output scenario
  - Third stage: the worst-case price-elastic demand curve

$$\max_{y,u,v} -\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}(SU_i^b u_{it}^b + SD_i^b v_{it}^b) + \min_{w\in\mathcal{W}} \max_{x,h\in\mathcal{X}} \Big(\min_{\epsilon\in\Pi} \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{k=1}^{K}(p_t^{bk*} + \epsilon_t^{bk})h_t^{bk} - \sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{i=1}^{G_b}\phi_{it}^b\Big)$$

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& \qquad d_t^b = \textstyle\sum_{k=1}^{K} h_t^{bk},\ 0 \leq h_t^{bk} \leq l_t^{bk}.\\
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\end{aligned}$$

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  - Third stage: the worst-case price-elastic demand curve

$$\max_{y,u,v} \ (\hat{a}^T u + \hat{b}^T v) + \min_{w \in \mathcal{W}} \max_{x,h,\phi \in \chi} (\min_{\epsilon \in \prod} \epsilon^T h + \hat{c}^T h - \hat{e}^T \phi)$$

$$\text{s.t.} \quad \hat{A}y + \hat{B}u + \hat{C}v \leq 0, y, u, v \in \{0, 1\}$$

$$\mathcal{X} = \Big\{ \hat{D}x \leq \hat{F}y + \hat{g},$$

$$\hat{P}x - \hat{J}\phi \leq \hat{K}y, \ \hat{R}h \leq \hat{q},$$

$$\hat{T}x + \hat{S}h \leq \hat{s} + \hat{O}w, x, h \geq 0 \Big\}$$

$$\Pi = \Big\{ \hat{U}\epsilon \geq \hat{m} \Big\}$$

Bender's Decomposition

- Dualize the constraints in Π and combine them with the second stage constraints

$$
\begin{aligned}
\max_{y,u,v}\ & (\hat{a}^T u + \hat{b}^T v) + \min_{w \in \mathcal{W}} \max_{x,h,\phi,\theta \in \bar{\chi}} (\hat{m}^T \theta + \hat{c}^T h - \hat{e}^T \phi) \\
\text{s.t.}\ & \hat{A} y + \hat{B} u + \hat{C} v \leq 0, \\
& y, u, v \in \{0, 1\} \\
& \bar{\chi} = \chi \cap \left\{ \hat{U}^T \theta - h \leq 0, \theta \geq 0 \right\}
\end{aligned}
$$

Bender's Decomposition

- Dualize the remaining constraints in $\bar{\chi}$

$$\omega(y) = \min_{w\in\mathcal{W},\gamma,\lambda,\tau,\mu,\eta} (\hat{F}y+\hat{g})^T\gamma + (\hat{K}y)^T\lambda + \hat{q}^T\tau + (\hat{s}+\hat{O}w)^T\mu$$

$$\text{s.t.} \quad \hat{D}^T\gamma + \hat{P}^T\lambda + \hat{T}^T\mu \geq 0,\ \hat{J}^T\lambda = \hat{e},$$

$$\hat{R}^T\tau + \hat{S}^T\mu - \eta \geq \hat{c},\ \hat{U}\eta \geq \hat{m},$$

$$\gamma,\lambda,\tau,\mu,\eta \geq 0.$$

Bilinear Terms

- Replace the bilinear terms: $w^T \hat{O}^T \mu$
  - Let $\hat{O}^T \mu = \sigma$
  - Using the uncertainty set W

$$
\begin{aligned}
w^T \sigma &= \sum_{t=1}^{T} \sum_{b=1}^{B} \sigma_t^b w_t^b \\
&= \sum_{t=1}^{T} \sum_{b=1}^{B} \sigma_t^b (W_t^{b*} + z_t^{b+} W_t^{b+} - z_t^{b-} W_t^{b-}) \\
&= \sum_{t=1}^{T} \sum_{b=1}^{B} (\sigma_t^b W_t^{b*} + \sigma_t^{b+} W_t^{b+} + \sigma_t^{b-} W_t^{b-})
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & \sigma_t^b = (\hat{O}^T \mu)_t^b, \\
& \sigma_t^{b+} \geq -M z_t^{b+}, \sigma_t^{b+} \geq \sigma_t^b - M(1 - z_t^{b+}), \\
& \sigma_t^{b-} \geq -M z_t^{b-}, \sigma_t^{b-} \geq -\sigma_t^b - M(1 - z_t^{b-}), \\
& \textstyle\sum_{t=1}^{T} (z_t^{b+} + z_t^{b-}) \leq \pi^b. \\
& (t = 1, 2, \cdots, T, b = 1, 2, \cdots, B)
\end{aligned}
$$

Master Problem

- Use the Benders' decomposition algorithm to solve the three-stage robust optimization problem.
- Denote θ as the second-stage optimal objective value.
- Add feasibility and optimality cuts to solve the problem iteratively.

Master problem

$$
\begin{aligned}
\omega^{\mathcal{P}} = \max_{y,u,v}\ & \hat{a}^T u + \hat{b}^T v + \theta \\
\text{s.t.}\quad & \hat{A}y + \hat{B}u + \hat{C}v \leq 0, \\
& \text{Feasibility cuts}, \\
& \text{Optimality cuts}, \\
& y, u, v \in \{0, 1\}.
\end{aligned}
$$

Feasibility Cuts

- Infeasibility detection: use L-shaped method to detect infeasibility and generate feasibility cuts:
  - If $\omega^f(y) = 0$, y is feasible.
  - If $\omega^f(y) < 0$, generate a feasibility cut $\omega^f(y) \geq 0$.

Feasibility detection

$$\omega^f(y) = \min_{w \in \mathcal{W}, \hat{\gamma}, \hat{\tau}, \hat{\mu}, \hat{\eta}} (\hat{F}y + \hat{g})^T \hat{\gamma} + \hat{q}^T \hat{\tau} + \hat{s}^T \hat{\mu} + (W^*)^T \hat{\sigma} + (W^+)^T \hat{\sigma}^+ + (W^-)^T \hat{\sigma}^-$$

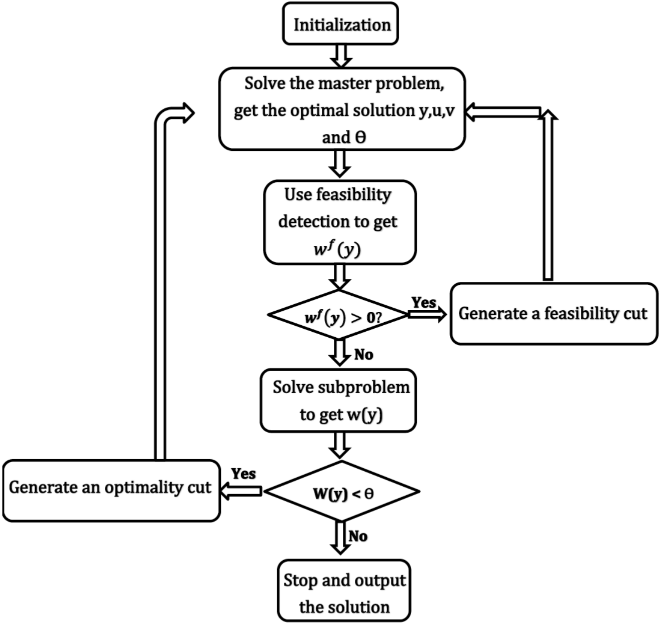
$$\begin{aligned} \text{s.t.} \quad & \hat{D}^T \hat{\gamma} + \hat{T}^T \hat{\mu} \geq 0, \\ & \hat{R}^T \hat{\tau} + \hat{S}^T \hat{\mu} \geq 0, \\ & \hat{M}_1^T \hat{\sigma} + \hat{M}_2^T \hat{\sigma}^+ + \hat{M}_3^T \hat{\sigma}^- + \hat{N}^T z \leq \hat{X}, \\ & \hat{\gamma}, \hat{\tau}, \hat{\mu}, \hat{\sigma}, \hat{\sigma}^+, \hat{\sigma}^- \in [0, 1] \end{aligned}$$

Optimality Cuts

- Solve the master problem to get θ
- If $\theta > \omega(y)$, generate an optimality cut $\theta \leq \omega(y)$

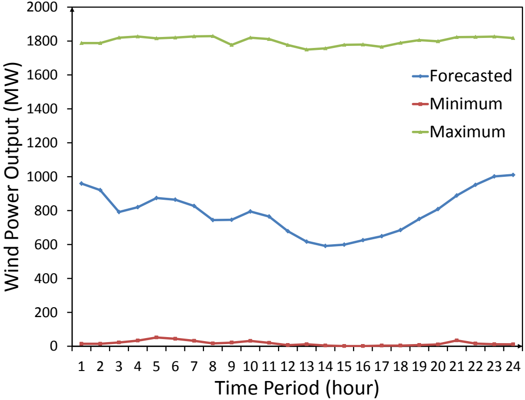
$$\begin{aligned}
\omega(y) = \min_{w \in \mathcal{W}, \gamma, \lambda, \tau, \mu, \eta} & (\hat{F}y + \hat{g})^T \gamma + (\hat{K}y)^T \lambda + \hat{q}^T \tau + (\hat{s} + \hat{O}w)^T \mu \\
\text{s.t.} \quad & \hat{D}^T \gamma + \hat{P}^T \lambda + \hat{T}^T \mu \geq 0, \ \hat{J}^T \lambda = \hat{e}, \\
& \hat{R}^T \tau + \hat{S}^T \mu - \eta \geq \hat{c}, \ \hat{U} \eta \geq \hat{m}, \\
& \gamma, \lambda, \tau, \mu, \eta \geq 0.
\end{aligned}$$

Algorithm Scheme



Computational Experiments

- ILOG CPLEX 12.1, Intel Quad Core 2.40GB, 8GB memory
- 118 Buses, 186 transmission lines, 24 time period
- The pattern of the wind power output is based on the statistics from National Renewable Energy Laboratory (NREL).



Effectiveness of Demand Response

- Without demand response (CD) vs with deterministic demand response (DR).

π	System	ULC	Time(s)	Cuts
2	CD	12.518	320	11
	DR	12.239	579	10
4	CD	12.624	413	15
	DR	12.368	713	11
6	CD	12.722	893	14
	DR	12.455	1005	13
8	CD	12.814	1126	14
	DR	12.553	1318	14

- Under the worst-case wind power output scenario, the case with demand response has less unit load cost than the case without demand response given the same wind cardinality budget π.

Wind Power Output and Demand Response Uncertainties

- Both wind power output and demand response uncertainties.

π	ϵ	ULC	S.W.(e+6)	Consumer (e+6)	Supplier(e+6)
2	0	12.239	7.03246	4.09943	2.93303
	5	12.205	6.51834	4.10654	2.4118
	10	12.002	6.01348	4.11373	1.89975
4	0	12.368	7.01773	4.08626	2.93147
	5	12.339	6.50367	4.09272	2.41095
	10	12.142	5.99897	4.10004	1.89893
6	0	12.445	7.00967	4.07826	2.93141
	5	12.416	6.49478	4.08475	2.41003
	10	12.221	5.99113	4.09229	1.89884

- π increases $\rightarrow$ ULC increases and both consumer surplus and supplier surplus decrease.
- more demand response uncertainty $\rightarrow$ lower total social welfare.

Multiple Wind Farm System

- Distributed wind resources– wind farms at buses 10, 26, and 32.

π	System	ULC	Time(s)	Cuts
1	CD	12.9906	993	12
	DR	12.8021	1272	9
2	CD	13.0588	1533	14
	DR	12.8335	2086	9
4	CD	13.1336	2875	13
	DR	12.9045	3408	7

- With multiple wind resources, the case with demand response still has smaller unit load cost than the case without demand response given the same wind cardinality budget.
- The CPU time increases dramatically as the number of wind farms increases.

Conclusion

- The proposed multi-stage robust integer programming approach can accommodate both wind power and demand response uncertainties.
- Demand response can help accommodate wind power output uncertainty by lowering the unit load cost.

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