Algorithms for Solving Stochastic Unit Commitment and Security-Constrained Economic Dispatch Problems

Dzung Phan, Ali Koc, and Jayant Kalagnanam
IBM T.J. Watson Research Center

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Outline

- Stochastic Unit Commitment
  - Scenario reduction and scenario tree generation
  - Parallel branch-cut-price

- Optimal Power Flow
  - Algorithms for security-constrained OPF
  - OPF under uncertainty

Stochastic Unit Commitment

UC Formulation

Indices and sets:

$i \in I$	generators
$t \in \{1, \ldots, T\}$	time periods

Data:

$S_i(\cdot)$	startup cost function of unit i
$H_i(\cdot)$	shutdown cost function of unit i
$f_i(\cdot)$	generation cost function of unit i
(Q_i, q_i)	maximum and minimum amounts for units i's offer
(R_i, r_i)	ramp-up and ramp-down amounts for unit i
(L_i, l_i)	minimum up and down times for unit i
d_t	load at time period t

Decision variables:

g_{it}	generation provided by unit i at time period t
u_{it}	binary variable indicating if unit i is on at time period t
$\overline{u}_{it}$	vector of u_{it} variables. $\overline{u}_{it} \equiv (u_{i\tau})$, $\tau = t - l_i + 1, \ldots, t$.

UC Formulation

Formulation:

$$\min_{\mathbf{g},\mathbf{u}} \sum_{t=1}^{T} \sum_{i \in I} S_i(\overline{u}_{it}) + H_i(\overline{u}_{it}) + f_i(g_{it}) \tag{1a}$$

$$\text{s.t.} \sum_{i \in I} g_{it} \geq d_t, \qquad t = 1, \ldots, T, \tag{1b}$$

$$q_i u_{it} \leq g_{it} \leq Q_i u_{it}, \qquad i \in I,\ t = 1, \ldots, T, \tag{1c}$$

$$g_{it} - g_{i,t-1} \leq R_i, \qquad i \in I,\ t = 1, \ldots, T, \tag{1d}$$

$$g_{i,t-1} - g_{it} \leq r_i, \qquad i \in I,\ t = 1, \ldots, T, \tag{1e}$$

$$u_{it} - u_{i,t-1} \leq u_{i\tau}, \qquad \tau \in \{t+1, \ldots, \min\{t + L_i, T\}\},\ i \in I,\ t = 1, \ldots, T, \tag{1f}$$

$$u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, \qquad \tau \in \{t+1, \ldots, \min\{t + l_i, T\}\},\ i \in I,\ t = 1, \ldots, T, \tag{1g}$$

$$u_{it} \in \{0, 1\},\ g_{it} \geq 0, \qquad i \in I,\ t = 1, \ldots, T. \tag{1h}$$

UC Formulation Extensions

Spinning reserves:

$$\sum_{i \in I} o_{it} > O_t^S, \qquad t = 1, \ldots, T,$$

$$o_{it} + g_{it} - g_{i,t-1} \leq R_i, \qquad i \in I,\ t = 1, \ldots, T,$$

$$o_{i,t-1} + g_{i,t-1} - g_{it} \leq r_i, \qquad i \in I,\ t = 1, \ldots, T,$$

$$o_{it} + g_{it} \leq Q_i u_{it}, \qquad i \in I,\ t = 1, \ldots, T,$$

$$o_{it} > 0, \qquad i \in I,\ t = 1, \ldots, T,$$

Non-spinning reserves:

$$\sum_{i \in I} g_{it} + \rho . v_{t-1} - v_t \geq d_t, \qquad t = 1, \ldots, T,$$

$$v_t > 0,\ \ v_0 = 0, \qquad t = 1, \ldots, T,$$

Spot market buy/sell / Demand response

$$\min_{\ldots, \mathbf{g}, \mathbf{m}^+, \mathbf{m}^-, \ldots} \quad \ldots + \sum_{t=1}^{T} (p^+ m_t^+ - p^- m_t^-) + \ldots$$

$$\text{s.t.} \quad \sum_{i \in I} g_{it} + m_t^+ - m_t^- \geq d_t, \qquad t = 1, \ldots, T,$$

$$m_t^+ > 0,\ \ m_t^- > 0, \qquad t = 1, \ldots, T,$$

$$\ldots\ldots\ldots \qquad \ldots$$

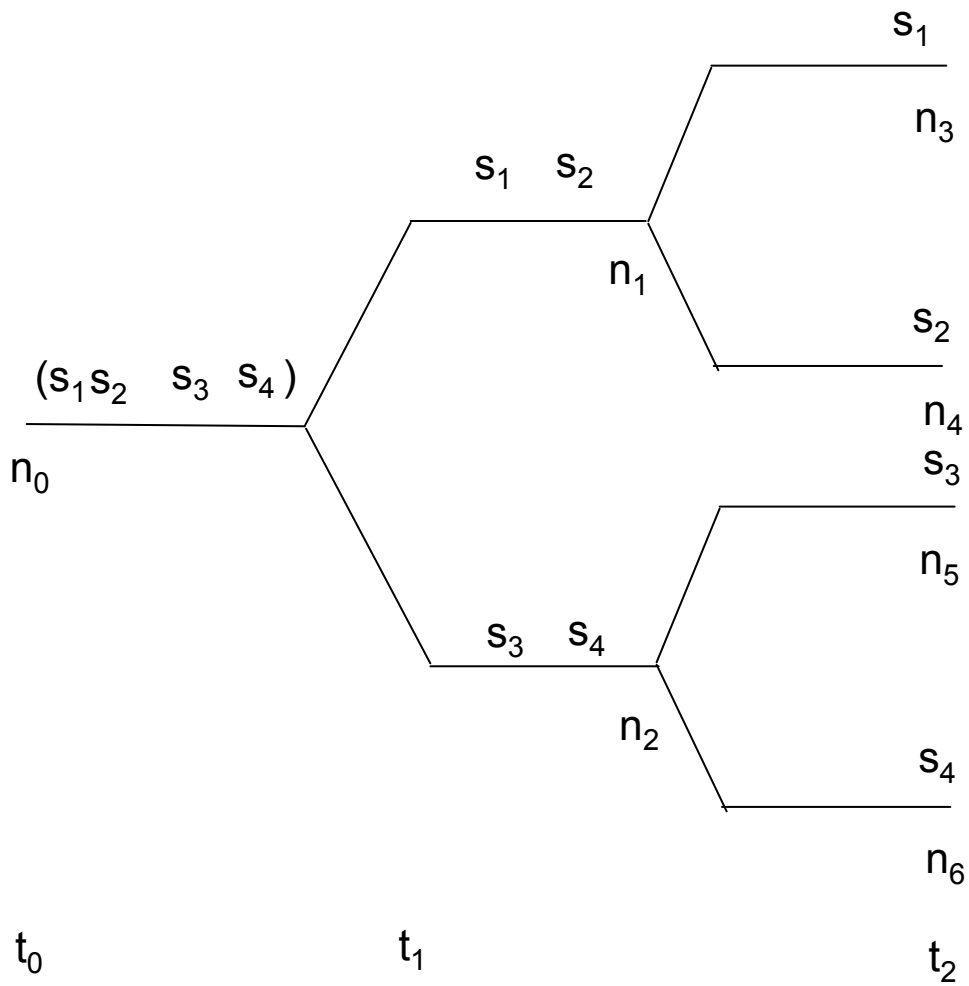
Storage

$$\sum_{i \in I_P} Q_i - g_{it} > O_t^N, \qquad t = 1, \ldots, T,$$

DC power flow constraints

Underway ……

Stochastic Unit Commitment – Scenario Tree



Scenario Formulation:
If two scenarios are indistinguishable up to time *t*, then the decisions for both scenarios by time *t* should be the same.
If $d_t^{s_1} = d_t^{s_2}\ \forall t < \tau$ then we add equalities $u_{i,\tau}^{s_1} = u_{i,\tau}^{s_2}\ \forall i$

We call the collection of this equalities as nonanticipativity constraints

Nodal Formulation:
We use a single set of decision variables for each node of the scenario tree.

Stochastic Unit Commitment - Formulation

$$
\begin{aligned}
\min_{\mathbf{g},\mathbf{u},\mathbf{s}} \quad & \sum_{n\in N} p_n \sum_{i\in I} S_i(\overline{u}_{in}) + H_i(\overline{u}_{in}) + f_i(g_{in}) & \\
\text{s.t.} \quad & \sum_{i\in I} g_{in} \geq d_n, & n \in N, \\
& q_i u_{in} \leq g_{in} \leq Q_i u_{in}, & i \in I,\ n \in N \\
& s_{in} \geq u_{in} - u_{i,n(1)}, & i \in I,\ n \in N \\
& g_{in} - g_{i,n(1)} \leq R_i, & i \in I,\ n \in N \\
& g_{i,n(1)} - g_{in} \leq r_i, & i \in I,\ n \in N \\
& \sum_{\tau=0}^{L_i-1} s_{i,n(\tau)} \leq u_{in}, & i \in I,\ n \in N \\
& \sum_{\tau=1}^{l_i-1} s_{i,n(\tau)} \leq 1 - u_{i,n(l_i)}, & i \in I,\ n : n(l_i) \in N \\
& \sum_{\tau=1}^{l_i-1-p} s_{i,n(\tau)} \leq 1 - u_{i,n(l_i-p)}, & i \in I,\ n \in N_L,\ p \in \{1,\ldots,l_i-1\} \\
& u_{in} \in \{0,1\},\ s_{in} \in [0,1],\ g_{in} \geq 0 &
\end{aligned}
$$

Integrating Renewables

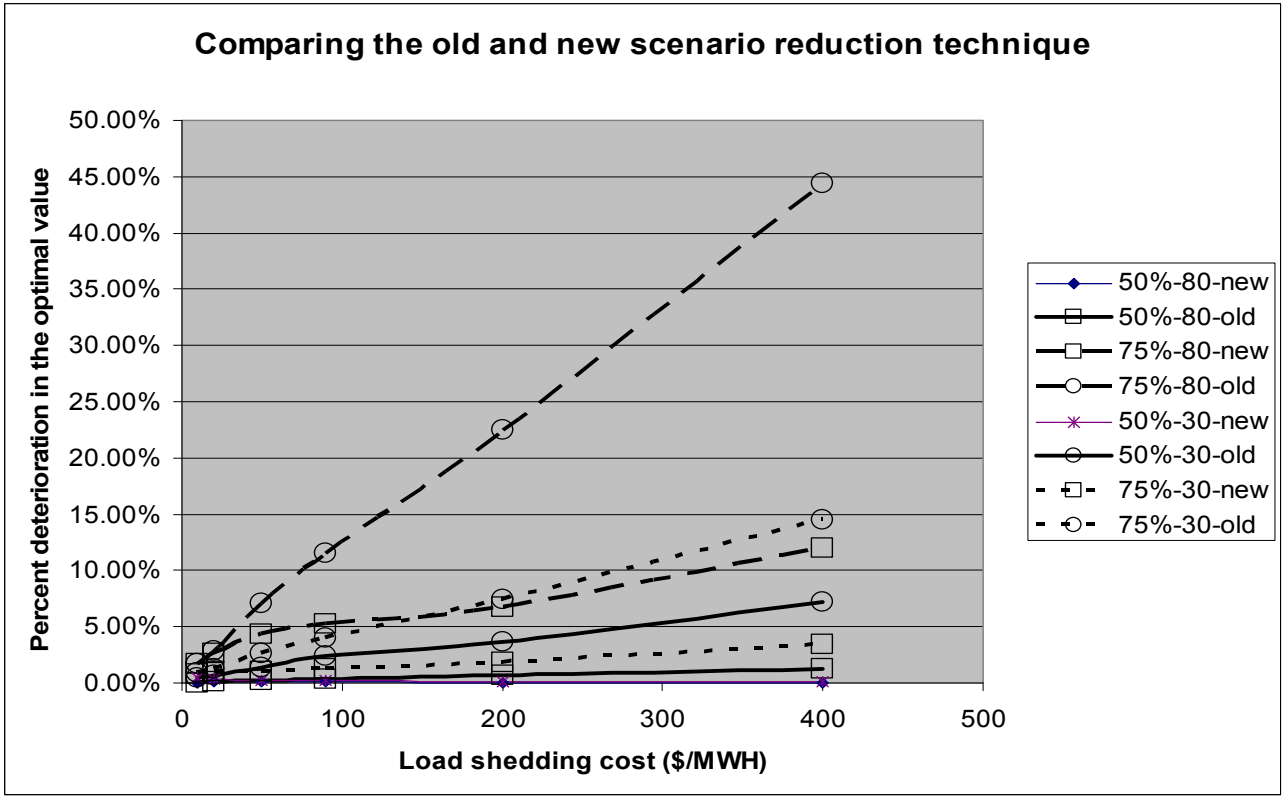
- Wind energy is forecasted using weather models
  - Wind speed and direction can be forecasted but with uncertainty
  - For each farm, generation $g_{i,t}$ is a random variable
- Assume that wind energy (subject to technical cut-in constraints) has to be used (regulatory)
  - A must-take constraint
- Therefore the total demand can be written as
  - $D_t = d_t - \Sigma_i g_{i,t}$ (a new random variable)

- In DC power flow model, each wind farm acts as a new generation demand point, and thus modeled separately.

Scenario Reduction

- ***Objective of scenario reduction:*** $\min F(\tilde{x}) - F(x^*)$

- Establishes a tighter upper bound than Pflug's approach, which considers the optimization problem underlying the stochastic program in addition to the Pflug's approach that considers only the underlying stochastic process.
- Not only works with the worst-case upper bound, but also obtains the real upper bound asymptotically.



Scenario Tree Generation

- $X_t = \alpha X_{t-1} + \varepsilon_t$ -AR(1) model, t = 1,…, 4

<u>SAA Approach</u>

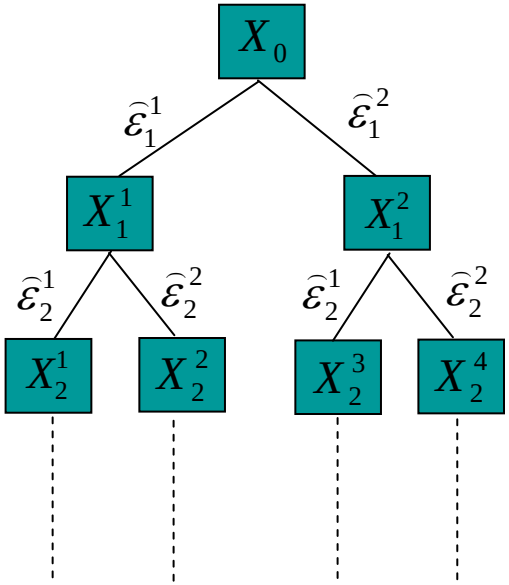
- Sample from (ε_1, ε_2, ε_3, ε_4) and solve the sample average approximation problem

<u>SAA & Reduction Approach</u>

- Sample from (ε_1, ε_2, ε_3, ε_4), and do scenario reduction over all sample paths

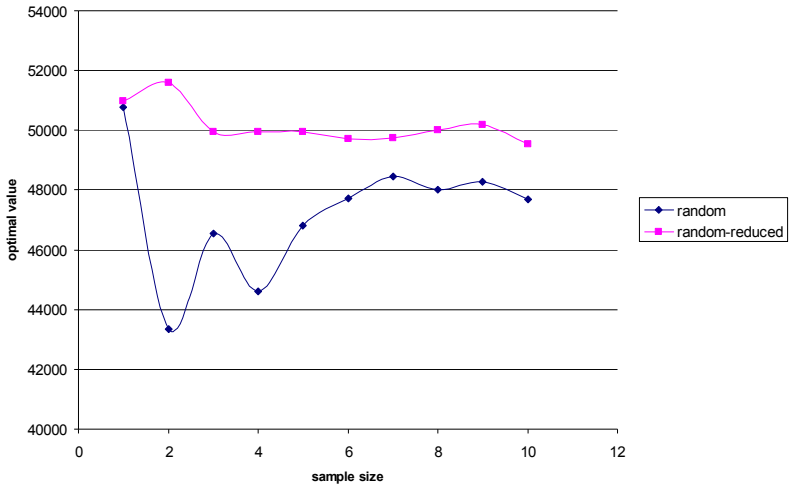
<u>Our approch</u>

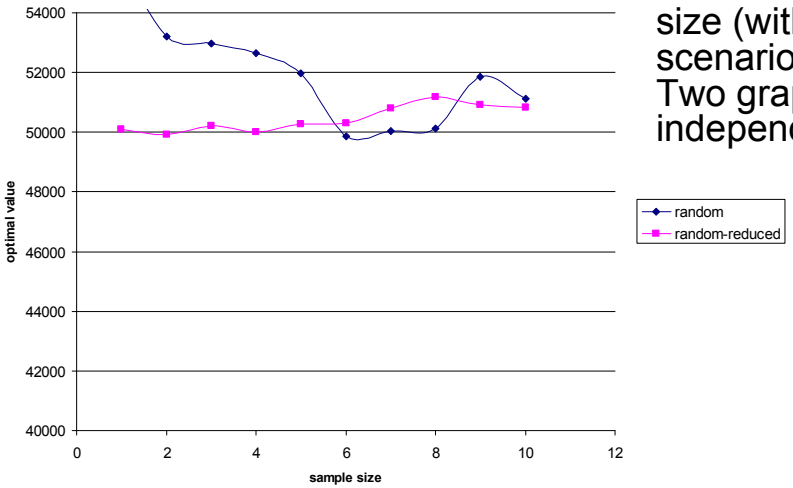
- Generate representative mass points from (ε_1, ε_2, ε_3, ε_4)
- Sample from ε_1 and do scenario reduction
  sample from ε_2 and do scenario reduction
  ……..
- The advantage is less scenario-reduction effort.
- Proven that under certain conditions, this approach and SAA & Reduction approach are equivalent.



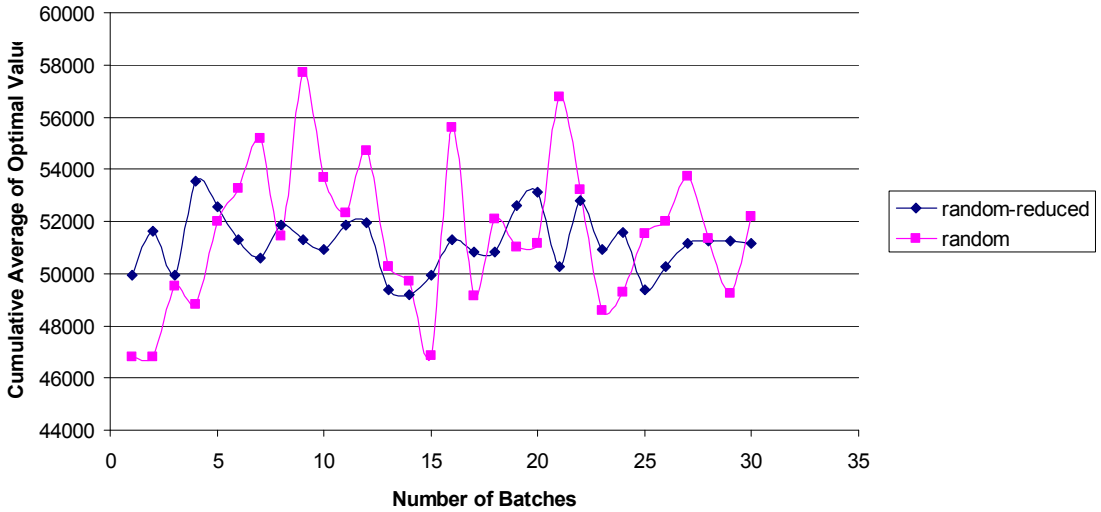
Scenario Tree Generation

- $X_t = X_{t-1} + \text{Normal}(0,5)$ -AR(1) model, t = 1,…, 4





Optimal value of SAA problem for increasing sample size (with and without scenario reduction). Two graphs for two independent batches



Cumulative average of optimal values of SAA problem over thirty batches for sample sizes of three (with and without scenario reduction)

Branch-Cut-Price Decomposition

- We use COIN-BCP, to implement branch-cut-price, COIN-CGL to generate valid inequalities, COIN-OSI and Cplex to solve the master problem and some of the pricing problems, and dynamic programming to solve the pricing problems.
- We use four numbers to code a problem instance. First two numbers represent the scenario tree, third number represents the number of units, and last number represents the initial status of units. 24-111-1536-2, for example, means an instance with 24 time periods, 111 scenarios, 1536 units, and second set of initial statuses.
- We run the serial algorithm on an Intel Xeon E5410 processor with 2.33 GHz speed and 4 GB memory. We run the parallel algorithm on a 32-processor cluster arranged in four groups of eight SMP processors of this type. We solve the problems to a 0.01% optimality gap, unless otherwise stated, and report the solution times in CPU seconds.
- We test several strategies
  - Comparing decompositions
  - Weighted column generation
  - Combining with Lagrangian relaxation
  - Stabilization
  - Primal heuristics
  - Valid Inequalities

Branch-Cut-Price vs. Cplex

quadratic objective			linear objective		
Problem Instance	BCP	Cplex	Problem Instance	BCP	Cplex
6-30-323	17	2.08%	6-30-32-1	45	10
6-30-96-3	128	16.59%	6-30-96-1	137	82
6-30-192-3	340	18.46%	6-30-192-1	187	205
6-30-384-3	508	na	6-30-384-1	396	1848
7-30-32-3	108	6.34%	6-30-768-1	526	22K
9-30-32-3	0.02%	12.85%	20-20-32-1	1258	150
12-30-32-3	0.13%	23.50%	20-20-96-1	2985	1515
12-80-32-3	0.22%	3.20%	20-20-192-1	4498	7040
12-80-96-3	0.24%	na	20-20-384-1	10388	25K
12-80-192-3	0.23%	na	20-20-768-1	18391	132K
12-80-384-3	0.32%	na	12-80-32-1	1790	183
12-80-768-3	0.39%	na	12-80-96-1	3590	1949
20-20-32-3	1.62%	na	12-80-192-1	6322	8015
20-20-96-3	0.54%	na	12-80-384-1	16764	53K
20-30-32-3	1.89%	1.76%	12-80-768-1	23441	263K
20-40-32-3	1.95%	na			
20-80-96-3	0.23%	na			
24-111-32-3	1.50%	na			
24-221-32-3	1.96%	na			

Note: We solve ISO model instances with linear and quadratic fuel cost functions. In instances with quadratic cost, we report running time of the algorithm if it terminates within one hour, or remaining optimality gaps if it does not. In instances with linear fuel cost, we run the algorithm until a solution within 0.01% optimality is obtained.

Intra-Node Parallelization

		speedup with load balancing					w/o load balancing
Prob. inst.	ser'l time	2	4	8	16	32	32
6-30-32-3	147	1.90	3.58	6.58	10.83	17.77	14.92
6-30-96-3	455	1.91	3.75	7.12	11.07	19.76	15.81
6-30-192-3	587	1.93	3.78	7.29	12.07	19.65	16.19
6-30-384-3	948	1.95	3.85	7.45	12.98	20.50	16.43
7-30-32-3	200	1.65	3.25	5.57	10.14	16.72	14.32
9-30-32-3	322	1.70	3.68	6.04	10.24	17.59	13.91
12-30-32-3	488	1.94	3.69	6.50	11.73	17.79	14.69
12-80-32-3	1054	1.83	3.62	6.27	10.68	18.52	15.48
12-80-96-3	3301	1.85	3.70	6.66	12.79	22.78	18.78
12-80-192-3	6943	1.89	3.71	7.67	14.31	24.03	19.22
12-80-384-3	13505	1.90	3.73	7.81	15.24	23.75	19.72
12-80-768-3	27169	1.91	3.80	7.89	15.58	23.68	18.82
20-20-32-3	277	1.83	3.60	6.68	12.95	19.27	16.05
20-20-96-3	1734	1.85	3.66	7.20	13.27	22.64	18.21
20-30-32-3	542	1.85	3.65	7.82	15.74	20.42	16.05
20-40-32-3	877	1.82	3.59	7.80	15.44	20.32	16.81
20-80-96-3	7200	1.89	3.66	7.83	15.64	22.88	17.14
24-111-32-3	4983	1.66	3.45	6.81	12.72	17.65	14.13
24-221-32-3	27126	1.64	3.15	5.83	11.14	14.06	11.23
Average		**1.83**	**3.62**	**6.95**	**12.73**	**19.80**	**16.06**

Note: We solve utility model with quadratic cost and with buying and selling costs of \$40 and \$15 per megawatt-hour. The problems are solved to within 0.01% optimality.

Hybrid Parallelization

		speedup				
Prob. inst.	# nodes	(1,29)	(10,20)	(15,15)	(20,10)	(29,1)
72-3-32-3	9	14.82	15.26	12.04	9.31	4.09
72-3-96-3	12	16.81	18.75	16.12	11.07	5.76
72-3-192-3	10	17.58	18.78	15.29	12.09	5.65
72-3-384-3	8	20.52	20.85	17.45	12.98	6.50
72-3-768-3	13	19.48	21.25	18.57	11.14	6.72
72-6-32-3	98	13.70	15.58	16.58	12.83	9.77
72-6-96-3	104	15.31	16.75	15.12	14.07	10.76
72-6-192-3	100	15.93	17.78	19.29	15.23	9.65
72-6-384-3	119	18.85	19.85	21.45	16.48	11.50
72-6-768-3	115	18.65	21.25	23.57	16.14	11.72
72-10-32-3	178	12.30	17.58	22.58	18.83	12.77
72-10-96-3	192	14.41	18.75	23.12	19.07	11.76
72-10-192-3	155	17.23	19.78	23.29	18.07	12.65
72-10-384-3	201	18.75	20.85	24.45	16.98	10.50
72-10-768-3	182	19.65	19.25	23.57	16.14	12.72

Note: We solve ISO model with linear fuel cost function to within 0.1% optimality. Excluding the two master processors, a total of 30 processors are used for each experiment. The two numbers in the parentheses denote the number of the first and the second types of slave processors respectively. For instance, (10,20) means we assign 10 of the 30 processors to the first group and 20 of them to the second group.

Solving Large Instances with Parallelization

	problem size			ISO model ('-1' instances)		utility model ('-3' instances)	
Problem Instance	# binaries	# continuous variables	# constraints ($\approx$)	gap (%)	speedup	solution time	speedup
24-111-32	32×1399	$2 \times 32 \times 1399$	$8 \times 32 \times 1399$	0.15	18.99	311	16.41
24-111-96	96×1399	$2 \times 96 \times 1399$	$8 \times 96 \times 1399$	0.05	17.19	1085	18.18
24-111-192	192×1399	$2 \times 192 \times 1399$	$8 \times 192 \times 1399$	0.15	19.49	1894	22.14
24-111-384	384×1399	$2 \times 384 \times 1399$	$8 \times 384 \times 1399$	0.08	20.37	3043	24.44
24-111-768	768×1399	$2 \times 768 \times 1399$	$8 \times 768 \times 1399$	0.08	22.15	4869	27.50
24-111-1536	1536×1399	$2 \times 1536 \times 1399$	$8 \times 1536 \times 1399$	0.11	24.43	7791	29.22
24-111-3072	3072×1399	$2 \times 3072 \times 1399$	$8 \times 3072 \times 1399$	1.25	25.28	17.7k	28.01
24-221-32	32×2774	$2 \times 32 \times 2774$	$8 \times 32 \times 2774$	0.29	17.82	2125	15.26
24-221-96	96×2774	$2 \times 96 \times 2774$	$8 \times 96 \times 2774$	0.15	18.28	5737	18.54
24-221-192	192×2774	$2 \times 192 \times 2774$	$8 \times 192 \times 2774$	0.32	18.93	1037	22.66
24-221-384	384×2774	$2 \times 384 \times 2774$	$8 \times 384 \times 2774$	0.24	19.59	1653	25.50
24-221-768	768×2774	$2 \times 768 \times 2774$	$8 \times 768 \times 2774$	0.32	21.50	26.7k	28.02
24-221-1536	1536×2774	$2 \times 1536 \times 2774$	$8 \times 1536 \times 2774$	0.27	23.98	42.3k	29.79
24-221-3072	3072×2774	$2 \times 3072 \times 2774$	$8 \times 3072 \times 2774$	1.82	24.09	68k	29.37

Note: Both models have linear fuel cost functions and are run on 32 processors. For ISO model, we run the algorithm for one hour, and for utility model we run it until we obtain a solution within 0.01% optimality.

Optimal Power Flow

- Security-Constrained OPF
- OPF under uncertainty

SCOPF Problem Formulation

- Security-constrained OPF: it minimizes a given objective function while ensuring that the operating constraints are satisfied in both the normal and all the contingency states

$\mathbf{min\ f(u_0)}$	Objective
$\mathbf{g_0(x_0, u_0) \leq 0}$	Normal case: pre-contingency operation
$\mathbf{g_c(x_c, u_0) \leq 0}$	Preventive: control variables **u** not rescheduled
$\mathbf{g_c(x_c, u_c) \leq 0}$	Corrective: reschedule **u** to remove post-contingency violation
$\mathbf{\lvert u_c - u_0 \rvert \leq \Delta_{max}}$	Ramp rates condition

- Major difficulties: large-scale, nonlinear, nonconvex, and possibly discrete variables
- Trying to solve this problem directly by imposing simultaneously all post-contingency constraints would lead to prohibitive memory requirements and CPU times

Solution Methods

- Branch-and-bound based on Lagrangian duality
  - Able to find the globally optimal solution
  - The lower bound is computed from Lagrangian duality, while the feasible set subdivision is obtained by rectangular or ellipsoidal bisection
- Benders decomposition
  - Decompose into a master problem and subproblems, where subproblems check solution feasibility for the master problem
  - Derive a new cut that exploits the special structure of problem
  - Handle the nonconvexity by the adaptive cuts
- Alternating direction methods of multipliers (ADMM)
  - A class of first-order primal-dual algorithms based on the augmented Lagrangian method
  - By introducing auxiliary variables for control variables, ADMM decomposes the original problem into a number of subproblems related to each contingency

Exact Methods with Rectangular Model

$$\min_{e_i^c, f_i^c, P_i^{g,c}, Q_i^{g,c}} \sum_{i \in \mathcal{G}} \left(c_{0i} (P_i^{g,0})^2 + c_{1i} P_i^{g,0} + c_{2i} \right)$$

subject to (for $c = 0, \ldots, C$)

$$\sum_{j \in \mathcal{N}} \left(e_i^c (g_{ij}^c e_j^c - b_{ij}^c f_j^c) + f_i^c (g_{ij}^c f_j^c + b_{ij}^c e_j^c) \right) = P_i^{g,c} - P_i^d, i \in \mathcal{N}$$

$$\sum_{j \in \mathcal{N}} \left(f_i^c (g_{ij}^c e_j^c - b_{ij}^c f_j^c) - e_i^c (g_{ij}^c f_j^c + b_{ij}^c e_j^c) \right) = Q_i^{g,c} - Q_i^d, i \in \mathcal{N}$$

$$((e_i^c)^2 + (f_i^c)^2) \bar{g}_{ii}^c + (e_i^c e_j^c + f_i^c f_j^c) \bar{g}_{ij}^c - (e_i^c f_j^c - e_j^c f_i^c) \bar{b}_{ij}^c \leq P_{ij}^{max,c}, (i,j) \in \mathcal{B}$$

$$f_1^c = 0$$

$$P_i^{min,c} \leq P_i^{g,c} \leq P_i^{max,c}, i \in \mathcal{G}$$

$$Q_i^{min,c} \leq Q_i^{g,c} \leq Q_i^{max,c}, i \in \mathcal{G}$$

$$(V_i^{min,c})^2 \leq (e_i^c)^2 + (f_i^c)^2 \leq (V_i^{max,c})^2, i \in \mathcal{N}$$

$$|P_i^{g,c} - P_i^{g,0}| \leq \Delta P_i^c, i \in \mathcal{G}$$

$$|Q_i^{g,c} - Q_i^{g,0}| \leq \Delta Q_i^c, i \in \mathcal{G},$$

Exact Methods with Rectangular Model

$$\min \quad f(\mathbf{P}^{g,0}) = \sum_{i \in \mathcal{G}} \left(c_{0i}(P_i^g)^2 + c_{1i}P_i^g + c_{2i} \right)$$

subject to (for $c = 0, \ldots, C$)

$$\sum_{j \in \mathcal{N}} \left(e_i^c(g_{ij}^c e_j^c - b_{ij}^c f_j^c) + f_i^c(g_{ij}^c f_j^c + b_{ij}^c e_j^c) \right) = P_i^{g,c} - P_i^d, i \in \mathcal{N}$$

$$\sum_{j \in \mathcal{N}} \left(f_i^c(g_{ij}^c e_j^c - b_{ij}^c f_j^c) - e_i^c(g_{ij}^c f_j^c + b_{ij}^c e_j^c) \right) = Q_i^{g,c} - Q_i^d, i \in \mathcal{N}$$

$$((e_i^c)^2 + (f_i^c)^2)\bar{g}_{ii}^c + (e_i^c e_j^c + f_i^c f_j^c)\bar{g}_{ij}^c - (e_i^c f_j^c - e_j^c f_i^c)\bar{b}_{ij}^c \leq P_{ij}^{max,c}, (i,j) \in \mathcal{B}$$

$$f_1^c = 0$$

$$P_i^{min,c} \leq P_i^{g,c} \leq P_i^{max,c}, i \in \mathcal{G}$$

$$Q_i^{min,c} \leq Q_i^{g,c} \leq Q_i^{max,c}, i \in \mathcal{G}$$

$$(V_i^{min,c})^2 \leq (e_i^c)^2 + (f_i^c)^2 \leq (V_i^{max,c})^2, i \in \mathcal{N}$$

$$|P_i^{g,c} - P_i^{g,0}| \leq \Delta P_i^c, i \in \mathcal{G}$$

$$|Q_i^{g,c} - Q_i^{g,0}| \leq \Delta Q_i^c, i \in \mathcal{G},$$

$$\sum_{i \in \mathcal{N}} ((e_i^c)^2 + (f_i^c)^2) \leq \sum_{i \in \mathcal{N}} (V_i^{max,c})^2, c = 0, \ldots, C.$$

Lagrangian Function

$$
\begin{aligned}
&\mathfrak{L}(\mathbf{e}^0, \mathbf{f}^0, \ldots, \mathbf{e}^C, \mathbf{f}^C, \mathbf{P}^{g,0}, \mathbf{Q}^{g,0}, \ldots, \mathbf{P}^{g,C}, \mathbf{Q}^{g,C}, \boldsymbol{\lambda}^1, \boldsymbol{\beta}^1, \boldsymbol{\mu}^1, \boldsymbol{\gamma}^1, \boldsymbol{\theta}^1, \ldots, \boldsymbol{\lambda}^C, \boldsymbol{\beta}^C, \boldsymbol{\mu}^C, \boldsymbol{\gamma}^C, \boldsymbol{\theta}^C) \\
&= f(\mathbf{P}^{g,0}) + \sum_{c=0}^{C} \Bigg\{ \sum_{i \in \mathcal{N}} \lambda_i^c \left(\sum_{j \in \mathcal{N}} (e_i^c (g_{ij}^c e_j^c - b_{ij}^c f_j^c) + f_i^c (g_{ij}^c f_j^c + b_{ij}^c e_j^c)) + P_i^d \right) + \\
&\quad \sum_{i \in \mathcal{N}} \beta_i^c \left(\sum_{j \in \mathcal{N}} (f_i^c (g_{ij}^c e_j^c - b_{ij}^c f_j^c) - e_i^c (g_{ij}^c f_j^c + b_{ij}^c e_j^c)) + Q_i^d \right) + \\
&\quad \sum_{(i,j) \in \mathcal{B}} \mu_{i,j}^c \left(((e_i^c)^2 + (f_i^c)^2) \bar{g}_{ii}^c + (e_i^c e_j^c + f_i^c f_j^c) \bar{g}_{ij}^c - (e_i^c f_j^c - e_j^c f_i^c) \bar{b}_{ij}^c \right) + \\
&\quad \sum_{i \in \mathcal{N}} \gamma_i^c \left((V_i^{min,c})^2 - (e_i^c)^2 - (f_i^c)^2 \right) + \sum_{i \in \mathcal{N}} \theta_i^c \left((e_i^c)^2 + (f_i^c)^2 - (V_i^{max,c})^2 \right) \\
&\quad - \sum_{i \in \mathcal{G}} \lambda_i^c P_i^{g,c} - \sum_{i \in \mathcal{G}} \beta_i^c Q_i^{g,c} \Bigg\}.
\end{aligned}
$$

Inner Subproblems

$$\min_{\mathbf{P}^{g,c},\mathbf{Q}^{g,c}:c=0,\ldots,C} \quad f(\mathbf{P}^{g,0}) - \sum_{c=0,\ldots,C}\left\{\sum_{i\in\mathcal{G}}\lambda_i^c P_i^{g,c} + \sum_{i\in\mathcal{G}}\beta_i^c Q_i^{g,c}\right\}$$

$$\begin{aligned}
\text{s.t.} \quad & P_i^{min,c} \le P_i^{g,c} \le P_i^{max,c}, \quad i\in\mathcal{G}, c=0,\ldots,C \\
& Q_i^{min,c} \le Q_i^{g,c} \le Q_i^{max,c}, \quad i\in\mathcal{G}, c=0,\ldots,C \\
& |P_i^{g,c} - P_i^{g,0}| \le \Delta P_i^c, \quad i\in\mathcal{G}, c=1,\ldots,C \\
& |Q_i^{g,c} - Q_i^{g,0}| \le \Delta Q_i^c, \quad i\in\mathcal{G}, c=1,\ldots,C
\end{aligned}$$

$$\begin{aligned}
\min_{\mathbf{e}^c,\mathbf{f}^c} \quad & \sum_{i\in\mathcal{N}}\lambda_i^c\left(\sum_{j\in\mathcal{N}}(e_i^c(g_{ij}^c e_j^c - b_{ij}^c f_j^c) + f_i^c(g_{ij}^c f_j^c + b_{ij}^c e_j^c))\right) + \\
& \sum_{i\in\mathcal{N}}\beta_i^c\left(\sum_{j\in\mathcal{N}}(f_i^c(g_{ij}^c e_j^c - b_{ij}^c f_j^c) - e_i^c(g_{ij}^c f_j^c + b_{ij}^c e_j^c))\right) + \\
& \sum_{(i,j)\in\mathcal{B}}\mu_{i,j}^c\Big(((e_i^c)^2 + (f_i^c)^2)\bar{g}_{ii}^c + (e_i^c e_j^c + f_i^c f_j^c)\bar{g}_{ij}^c - (e_i^c f_j^c \\
& -e_j^c f_i^c)\bar{b}_{ij}^c\Big) + \sum_{i\in\mathcal{N}}(\theta_i^c - \gamma_i^c)((e_i^c)^2 + (f_i^c)^2) \\
\text{s.t.} \quad & f_1^c = 0 \\
& (\mathbf{e}^c, \mathbf{f}^c) \in \mathcal{E}^c.
\end{aligned}$$

Benders Decomposition

SCOPF

$$\min_{\mathbf{x}_0,\ldots,\mathbf{x}_C,\mathbf{u}_0,\ldots,\mathbf{u}_C} \quad f_0(\mathbf{x}_0,\mathbf{u}_0)$$

$$\text{s.t.} \quad \mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0)=\mathbf{0}$$

$$\mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0)\le\mathbf{0}$$

$$\mathbf{g}_c(\mathbf{x}_c,\mathbf{u}_c)=\mathbf{0}$$

$$\mathbf{h}_c(\mathbf{x}_c,\mathbf{u}_c)\le\mathbf{0}$$

$$|\mathbf{u}_c-\mathbf{u}_0| \le \Delta\mathbf{u}_c^{\max}$$

Feasibility check

$$\mathbf{g}_c(\mathbf{x}_c,\mathbf{u}_c)=\mathbf{0}$$

$$\mathbf{h}_c(\mathbf{x}_c,\mathbf{u}_c)\le\mathbf{0}$$

$$|\mathbf{u}_c-\mathbf{u}_0| \le \Delta\mathbf{u}_c^{\max}$$

No

Cut Generation: $a^T x \le b$

Benders Decomposition

SCOPF

$$
\begin{aligned}
\min_{\mathbf{x}_0,\ldots,\mathbf{x}_C,\mathbf{u}_0,\ldots,\mathbf{u}_C} \quad & f_0(\mathbf{x}_0,\mathbf{u}_0) \\
\text{s.t.} \quad & \mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0)=\mathbf{0} \\
& \mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0)\le\mathbf{0} \\
& \\
& \mathbf{g}_c(\mathbf{x}_c,\mathbf{u}_c)=\mathbf{0} \\
& \mathbf{h}_c(\mathbf{x}_c,\mathbf{u}_c)\le\mathbf{0} \\
& |\mathbf{u}_c-\mathbf{u}_0| \le \Delta\mathbf{u}_c^{\max}
\end{aligned}
$$

Feasibility check

$$
\begin{aligned}
& G(y)\le 0 \\
& H(y)=0 \\
& A\bar{x}+By+b\le 0 \\
& y^L\le y\le y^U
\end{aligned}
$$

No

Cut Generation: $a^T x \le b$

Benders Decomposition

SCOPF

$$
\begin{aligned}
\min_{\mathbf{x}_0,\ldots,\mathbf{x}_C,\mathbf{u}_0,\ldots,\mathbf{u}_C} \quad & f_0(\mathbf{x}_0,\mathbf{u}_0) \\
\text{s.t.} \quad & \mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0)=\mathbf{0} \\
& \mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0)\le\mathbf{0} \\
& \\
& \mathbf{g}_c(\mathbf{x}_c,\mathbf{u}_c)=\mathbf{0} \\
& \mathbf{h}_c(\mathbf{x}_c,\mathbf{u}_c)\le\mathbf{0} \\
& |\mathbf{u}_c-\mathbf{u}_0| \le \Delta\mathbf{u}_c^{\max}
\end{aligned}
$$

Feasibility check

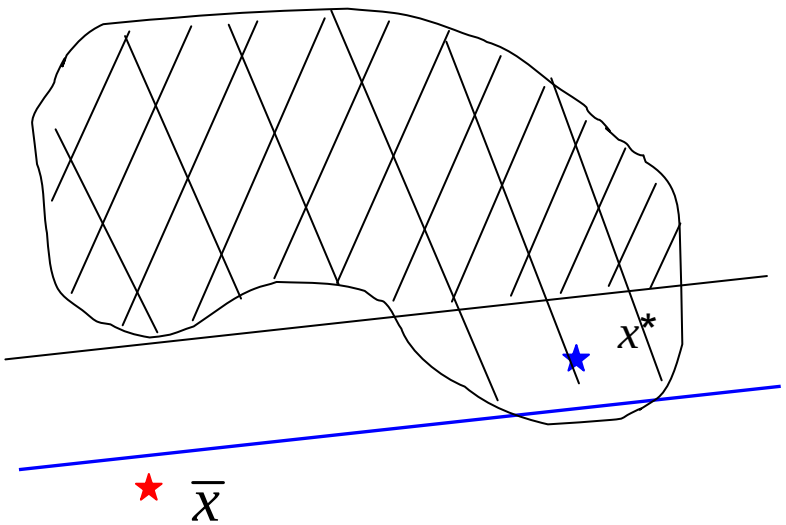
$$
\begin{aligned}
& \min_{\alpha,y}\sum_i \alpha_i \\
& G(y)\le 0 \\
& H(y)=0 \\
& A\bar{x}+By+b-\alpha\le 0 \\
& y^L\le y\le y^U \\
& \alpha\ge 0
\end{aligned}
$$

No

Cut Generation: $a^T x \le b$

$$
\begin{aligned}
a &= A^T\pi \\
b &= -\pi^T(By^L+b)+[\eta^T\nabla_y H(\bar{y})+\mu^T\nabla_y G(\bar{y})](\bar{y}-y^L)
\end{aligned}
$$

Adaptive Benders Cut



- Suppose that $c(x)$ is a valid cut for $\bar{x}$, then

$$c(x) - \tau c(\bar{x}) \le 0$$

  is also a cut for any $0 \le \tau < 1$.
- τ is chosen by a constraint infeasibility measure for $\bar{x}$: the value is large, we select τ close to 1, otherwise τ is close to 0

- Speedup enhancement: Switch off a contingency c if it was feasible for incumbent $\bar{x}$
  - It is likely that no cuts are generated for later $\bar{x}$
  - At the end, we need to verify that final $\bar{x}$ is feasible for contingency c

ADMM for SCOPF

- ADMM is applicable to the problem with block separable structure

$$\min_{x,y} \ \{ f(x) + g(y) : Ax + By = b, x \in X, y \in Y \}$$

- ADMM algorithm:

$$\begin{aligned} x^{k+1} &= \arg\min_{x \in X} L_\beta(x, y^k, \lambda^k) \\ y^{k+1} &= \arg\min_{y \in Y} L_\beta(x^{k+1}, y, \lambda^k) \\ \lambda^{k+1} &= \lambda^k + \beta(Ax^{k+1} + By^{k+1} - b) \end{aligned}$$

where $L_\beta(x, y, \lambda) = f_0(\mathrm{x}_0, \mathrm{u}_0) + \sum_c \lambda_c^T (\mathrm{u}_0 - \mathrm{u}_{0c}) + \beta/2 \sum_c \| \mathrm{u}_0 - \mathrm{u}_{0c} \|^2$

ADMM for SCOPF

- We reformulate SCOPF by introducing slack variables $\mathbf{u}_{0c}$

$$
\begin{aligned}
\min_{\mathrm{x}_0,\ldots,\mathrm{x}_C,\mathbf{u}_0,\ldots,\mathbf{u}_C} \quad & f_0(\mathbf{x}_0,\mathbf{u}_0) \\
\text{s.t.} \quad & \mathbf{g}_0(\mathbf{x}_0,\mathbf{u}_0)=\mathbf{0} \\
& \mathbf{h}_0(\mathbf{x}_0,\mathbf{u}_0)\le\mathbf{0} \\
& \mathbf{g}_c(\mathbf{x}_c,\mathbf{u}_c)=\mathbf{0} \\
& \mathbf{h}_c(\mathbf{x}_c,\mathbf{u}_c)\le\mathbf{0} \\
& |\mathbf{u}_c-\mathbf{u}_{0c}|\le\Delta\mathbf{u}_c^{\max}, \quad c=1,\ldots,C \\
& \mathbf{u}_0-\mathbf{u}_{0c}=0, \qquad c=1,\ldots,C
\end{aligned}
$$

Let us consider

$$
\begin{aligned}
& f(x)=f_0(\mathrm{x}_0,\mathrm{u}_0), \quad g(y)=0 \\
& X=\{(\mathrm{x}_0,\mathrm{u}_0):\mathrm{g}_0(\mathrm{x}_0,\mathrm{u}_0)=0,\ \mathrm{h}_0(\mathrm{x}_0,\mathrm{u}_0)\le 0\} \\
& Y=\mathrm{U}_c\{(\mathrm{x}_c,\mathrm{u}_c,\mathrm{u}_{0c}):\mathrm{g}_c(\mathrm{x}_c,\mathrm{u}_c)=0,\ \mathrm{h}_c(\mathrm{x}_c,\mathrm{u}_c)\le 0, |\mathrm{u}_c-\mathrm{u}_{0c}|\le\Delta\mathrm{u}_c^{\max}\} \\
& Ax+By=b: \quad \mathrm{u}_0-\mathrm{u}_{0c}=0
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
(x_0^{k+1},u_0^{k+1}) &= \arg\min_{(x_0,u_0)\in X} \quad f_0(x_0,u_0)+\sum_c(\lambda_c^k)^T(u_0-u_{0c}^k)+\beta/2\sum_c\|u_0-u_{0c}^k\|^2 \\
(x_c^{k+1},u_c^{k+1},u_{0c}^{k+1}) &= \arg\min_{(x_c,u_c,u_{0c})\in Y_c} \quad (\lambda_c^k)^T(u_0^k-u_{0c})+\beta/2\|u_0^k-u_{0c}\|^2, \qquad c=1,\ldots,C \\
\lambda^{k+1} &= \lambda^k+\beta(Ax^{k+1}+By^{k+1}-b)
\end{aligned}
}
$$

Numerical Results in Serial Implementation

		Benders			Adaptive Benders		
Cases	Optimal	Cost	It	Time	Cost	It	Time
CH9	42.24	56.83	4	0.37	42.27	6	0.53
IEEE30	89.73	97.25	5	1.44	92.58	20	3.56
NE39	391.61	418.81	5	1.61	391.50	18	3.14
IEEE57	419.55	432.67	3	2.24	427.96	9	3.55
IEEE118	1333.04	1402.74	7	5.81	1332.92	18	12.63
IEEE300	7197.23	7325.25	5	12.97	7197.36	18	26.15
PL3012		26137.92	8	831.16	25923.62	12	955.08

		ADMM			Adaptive Benders		
Cases	Optimal	Cost	It	Time	Cost	It	Time
CH9	42.24	42.24	19	1.03	42.27	6	0.53
IEEE30	89.73	89.82	71	7.11	92.58	20	3.56
NE39	391.61	391.58	31	4.74	391.50	18	3.14
IEEE57	419.55	419.52	12	4.46	427.96	9	3.55
IEEE118	1333.04	1333.15	29	23.54	1332.92	18	12.63
IEEE300	7197.23	7197.41	16	24.61	7197.36	18	26.15
PL3012		25921.58	43	3270.52	25923.62	12	955.08

Estimated Running Time in Parallel Implementation

	Benders		Adaptive Benders		ADMM	
Cases	Ser. Time	Par. Time	Ser. Time	Par. Time	Ser. Time	Par. Time
CH9	0.37	0.13	0.53	0.18	1.03	0.32
IEEE30	1.44	0.16	3.56	0.42	7.11	1.01
NE39	1.61	0.17	3.14	0.32	4.74	0.55
IEEE57	2.24	0.20	3.55	0.38	4.46	0.41
IEEE118	5.81	0.26	12.63	0.44	23.54	0.79
IEEE300	12.97	0.47	26.15	0.82	24.61	0.74
PL3012	831.16	23.90	955.08	28.65	3270.52	106.29

Par.Time = total time of base case subproblems + efficiency factor*(total time of contingency subproblems)/number of processors

Stochastic OPF: Problem Formulation

- We model two stages of decisions separated by, say, 5 ~ 15 mins
- Second stage recourse provided by expensive, fast-response energy sources P^{in}, Q^{in} like spotmarket, peaker generators, demand response etc.
- The first stage of the two-stage problem

$$\min \quad f(P^g) + \sum_{\xi} p^{\xi}.\omega^{\xi}(PQ^g)$$

$$\text{s.t.} \quad \text{constraints for } 1st \text{ stage variables } PQ^g, V^{\xi}$$

- The 2nd stage of the two-stage problem

$$\omega^{\xi}(PQ^g) = \min \quad g(P^{in,\xi}) - h(P^{out,\xi})$$

$$\text{s.t.} \quad \text{constraints for } 2nd \text{ stage variables } PQ^{in,\xi}, PQ^{out,\xi}, V^{\xi}$$

Algorithms

- Assumption 1: For every $PQ^g \in [P_{min}, P_{max}] \times [Q_{min}, Q_{max}]$, the 2nd stage problem is feasible
- Assumption 2: The dual of 2nd stage problem has a zero duality gap
- We solve a sequence of lower approximations of the master problem by augmenting a piecewise-linear approximation of ω^{ξ}. Specifically, we generate and use a set of subgradient $\{\pi^{\xi}\}$ of ω^{ξ}.
- The *k*th iterate solves

$$
\begin{aligned}
\min \quad & f(P^g) + \sum_{\xi} p^{\xi} \eta^{\xi} \\
\text{s.t.} \quad & \text{constraints for } 1st \text{ stage variables } PQ^g, V^{\xi} \\
\text{and A:} \quad & \eta^{\xi} \geq \omega^{\xi}(PQ^{g,k}) + (\pi^{s,k})^T (PQ^g - PQ^{g,k}) \\
\text{B:} \quad & \sum_{\xi} p^{\xi} \eta^{\xi} \geq \sum_{\xi} p^{\xi} \omega^{\xi}(PQ^{g,k}) + \sum_{\xi} p^{\xi} (\pi^{s,k})^T (PQ^g - PQ^{g,k})
\end{aligned}
$$

Numerical Results

			Algorithm A		Algorithm B		ADMM	
Cases	Exp. cost	Gen. cost	iter	time	iter	time	iter	time
CH9	2142.9	2092.8	2	0.41	2	0.42	28	2.53
IEEE14	5735.4	5776.3	2	0.47	2	0.45	35	3.72
IEEE30	6284.1	6198.7	4	1.62	4	1.44	44	10.07
NE39	32465.4	32504.5	2	0.85	2	0.76	61	13.64
IEEE57	38306.6	38212.6	5	0.45	7	0.64	20	5.81
IEEE118	128576.7	128451.8	4	3.70	4	3.22	17	15.84
IEEE300	715032.6	714958.2	3	5.66	3	4.84	29	34.07
PL3012	2132712.7	2132825.6	4	198.0	4	127.8	31	702.2

- Our proposed outer-approximation algorithms converge quickly to the optima
- Theoretically, we can show that these decomposition schemes find the globally optimal solutions for the nonconvex optimization problems

Thank you for your attention!