Online Control of Cascading Power Failures

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Credits

DOE grant with I. Hiskens (Michigan), I. Dobson J. Linderoth, S. Wright (Wisconsin)

PhD students: Sean Harnett (Applied Math), A. Verma (O.R.)
Cascade cartoon

Load (demand)

Generator
Cascade cartoon
Cascade cartoon
Cascade cartoon
Cascade cartoon
Cascade cartoon

= lost demand
Formal cascade model (Dobson et al)

→ Initial fault event takes place (an “act of God”).
Formal cascade model (Dobson et al)

→ Initial fault event takes place (an “act of God”).

For $r = 1, 2, \ldots,$

1. Compute new power flows

2. Determine new set of outaged lines
More detailed cascade model

→ Initial fault event takes place (an “act of God”).
More detailed cascade model

→ Initial fault event takes place (an “act of God”).

For $r = 1, 2, \ldots$,

(round $r$ of cascade)

1. Reconfigure demands and generator output levels.
Islanding

Islanding supply > demand → proportional adjustment of generator outputs/loads → to be added: generator ramp-up (down?) rates, other limits

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Islanding

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For \( r = 1, 2, \ldots, \)

(round \( r \) of cascade)

1. Reconfigure demands and generator output levels.

2. New power flows are instantiated.
Which power flow model?

→ This talk: linearized flows
Which power flow model?

→ This talk: linearized flows

→ Working on: AC power flows
Which power flow model?

→ This talk: linearized flows

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Low (2011): some (all?) AC power flow problems can be (at least approximately) modeled using SDP
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▶ Efficient in the large scale setting?
Which power flow model?

→ This talk: linearized flows
→ Working on: AC power flows

Low (2011): some (all?) AC power flow problems can be (at least approximately) modeled using SDP

▶ Efficient in the large scale setting?
▶ However, “local” version seems workable – can either find a nearby solution, or **prove** that none exists (cannot be done with Newton-Raphson)
More detailed cascade model

→ Initial fault event takes place (an “act of God”).

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1. Reconfigure demands and generator output levels.
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(\text{round } r \text{ of cascade})

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2 New power flows are instantiated.
More detailed cascade model

→ Initial fault event takes place (an “act of God”).

For \( r = 1, 2, \ldots \),

(round \( r \) of cascade)

1. Reconfigure demands and generator output levels.

2. New power flows are instantiated.

3. The next set of line outages takes place. If none do, STOP
Outage mechanism

Notation: \( f^r_k \) = flow on line \( k \) in round \( r \)
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Set \( \tilde{f}_k^r = \alpha|f_k^r| + (1 - \alpha)\tilde{f}^{r-1}_k \), where \( 0 \leq \alpha \leq 1 \).
Outage mechanism

Notation: $f_k^r =$ flow on line $k$ in round $r$

Set $\tilde{f}_k^r = \alpha |f_k^r| + (1 - \alpha)\tilde{f}_k^{r-1}$, where $0 \leq \alpha \leq 1$.

( $\rightarrow \tilde{f}_k^r =$ running average of $|f_k|$)
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( \( \rightarrow \) running average of \( |f_k| \))

\( \rightarrow \) \( k \) outages if \( \tilde{f}^r_k > u_k \).
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( \( \tilde{f}^r_k \) = running average of \( |f^r_k| \))

\( \rightarrow \) \( k \) outages if \( \tilde{f}^r_k > u_k \). \( \text{or: } e \) outages if \( \tilde{f}^r_k \geq u_k \)
Controlled cascades

→ Initial outage event takes place.
Controlled cascades

→ Initial outage event takes place.  Compute control.
Controlled cascades

Initial outage event takes place. Compute control.

For $r = 1, 2, \ldots, R - 1$

1. Reconfigure demands and generator output levels.
Controlled cascades

→ Initial outage event takes place.  **Compute control.**

For \( r = 1, 2, \ldots, R - 1 \)

1. Reconfigure demands and generator output levels.

2. New power flows are instantiated.
Controlled cascades

→ Initial outage event takes place. **Compute control.**

For \( r = 1, 2, \ldots, R - 1 \)

1. Reconfigure demands and generator output levels.

2. New power flows are instantiated.

3a. **Take measurements and apply control to shed demand.**
Controlled cascades

→ Initial outage event takes place.  Compute control.

For  \( r = 1, 2, \ldots, R - 1 \)

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2. New power flows are instantiated.

3a. Take measurements and apply control to shed demand.

3b. Reconfigure generator outputs;
Controlled cascades

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For \( r = 1, 2, \ldots, R - 1 \)

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3a. Take measurements and apply control to shed demand.

3b. Reconfigure generator outputs; get new power flows.
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3a. Take measurements and apply control to shed demand.

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4. The next set of outages takes place.
Controlled cascades

→ Initial outage event takes place.  **Compute control.**

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1. Reconfigure demands and generator output levels.

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3a. **Take measurements and apply control to shed demand.**

3b. **Reconfigure generator outputs; get new power flows.**

4. The next set of outages takes place.

At round \( R \), reduce demands so as to remove any line overloads.
Adaptive affine controls

For each demand bus $v$, and round $r$, control triple $c_r^v$, $b_r^v$, $s_r^v$.
**Adaptive affine controls**

For each demand bus \( n \) \( v \), and round \( r \), control triple \( c^r_v, b^r_v, s^r_v \)

\[ \rightarrow \text{Parameterized by integers } r > 0 \text{ and } \delta > 0. \]
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\( \rightarrow \) Parameterized by integers \( r > 0 \) and \( \delta > 0 \).

At round \( r \),

\( \blacktriangleright \) Let \( \kappa^\delta = \max \text{ overload} \) of any line within radius \( \delta \) of \( \mathbf{v} \)
Adaptive affine controls

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→ Parameterized by integers \( r > 0 \) and \( \delta > 0 \).

At round \( r \),

- Let \( \kappa^\delta = \max \text{ overload} \) of any line within radius \( \delta \) of \( v \)
- If \( \kappa^\delta > c_r^v \), demand at \( v \) reduced (scaled) by a factor

\[
\min \left\{ 1, \left\{ b_r^v + s_r^v (c_r^v - \kappa^\delta) \right\}^+ \right\}.
\]
Adaptive affine controls

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\min \left\{ 1, \{ b^r_v + s^r_v (c^r_v - \kappa^\delta) \}^+ \right\}.
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Example: $(1, 1, s)$ control; scale $= \min \left\{ 1, \{ 1 + s (1 - \kappa) \}^+ \right\}$. 
Adaptive affine controls

For each demand bus $n$ $v$, and round $r$, control triple $c_v^r$, $b_v^r$, $s_v^r$

→ Parameterized by integers $r > 0$ and $\delta > 0$.

At round $r$,

- Let $\kappa^\delta = \max$ overload of any line within radius $\delta$ of $v$
- If $\kappa^\delta > c_v^r$, demand at $v$ reduced (scaled) by a factor

$$\min \left\{ 1, \left\{ b_v^r + s_v^r (c_v^r - \kappa^\delta) \right\}^+ \right\}.$$

Example: $(1, 1, s)$ control; scale $= \min \left\{ 1, \left\{ 1 + s (1 - \kappa) \right\}^+ \right\}$.

Problem: choose control so as to maximize demand at end of round $R$. 
General methodology: simulation-based optimization

Given a control vector $\tilde{u} = (c_r^v, b_r^v, s_r^v)$ (over all $v$ and $r$),

$\Upsilon(\tilde{u}) =$ total demand satisfied at cascade end (yield)

- Maximization of $\Upsilon(\tilde{u})$ should be (very?) fast
General methodology: simulation-based optimization

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(yield)

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- Optimization should be robust (noisy process)
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- From a strict perspective, $\Upsilon(\tilde{u})$ is not even continuous
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- From a strict perspective, $\Upsilon(\tilde{u})$ is not even continuous

$\Upsilon(\tilde{u})$ is obtained through a simulation
Derivative-free optimization

Conn, Scheinberg, Vicente, others

Rough description:

- Sample a number of control vectors $\tilde{u}$
- Use sample points to construct a convex approximation to $\Upsilon$
- Optimize this approximation; this yields a new sample point

Scalability to large dimensionality?
Derivative-free optimization

Conn, Scheinberg, Vicente, others

Rough description:

- Sample a number of control vectors $\tilde{u}$
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Scalability to large dimensionality? (Not?)
“First order” method

Given a control vector \( \tilde{u} \)

1. Estimate the “gradient” \( g = \nabla \Upsilon(\tilde{u}) \) through finite differences.
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 Requires $O(1)$ simulations per demand node.
“First order” method

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2. Estimate step size $\argmax \mathcal{Y}(\tilde{u} + \sigma g)$
“First order” method

Given a control vector $\tilde{u}$

1. Estimate the “gradient” $g = \nabla \Upsilon(\tilde{u})$ through finite differences.

   Requires $O(1)$ simulations per demand node.

2. Estimate step size $\arg\max \Upsilon(\tilde{u} + \sigma g)$

→ Easily parallelizable
Solving the optimal scaling problem, fast

For each \( r \), and for each component ("island") \( K \) at round \( r \),

\[
(c^r_s, b^r_s, s^r_s) = (c^r_t, b^r_t, s^r_t)
\]

for every \( s, t \) in \( K \)

Solving the optimal scaling problem, fast

For each $r$, and for each component ("island") $K$ at round $r$,

$$(c^r_s, b^r_s, s^r_s) = (c^r_t, b^r_t, s^r_t)$$

for every $s, t$ in $K$

Then, equivalent problem:

- In round $r$, choose $\alpha^r(K) \leq 1$ for each component $K$
- If bus $v \in$ component $K$, then scale its demand by $\alpha^r(K)$
Solving the optimal scaling problem

Notation:

- \( \hat{\beta} \) = supply/demand vector at time 0
- \( \Upsilon^R(\beta) \) = total demand using optimal control, at end of round \( R \), if the supply/demand vector is \( \beta \) at time 0
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- For each $t \geq 0$, compute $\Theta^R(t) \doteq \Upsilon^R(t\beta)$
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Theorem:

$\Theta^R(t)$ is piecewise linear nondecreasing with $O(m^{R-1}/R!) \text{ breakpoints.}$
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Actually, $O(f(R)m^2)$ breakpoints.

Well, probably $O(Rm)$ breakpoints.
Implementation

(1) Solve scaling problem

(2) Grid search around control found by scaling problem

(3) (optional) First-order method on demand-quantile control

(4) (optional) Then switch to first-order method
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- 24 cores (3 x 8-core Intel i7 systems)
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- Gurobi, Cplex used to solve linear systems
- (1) and (2) on grids with $10^4$s lines/buses require seconds
- Five gradient steps of general method: $\sim$ one hour wallclock
Experiments on the **Eastern Interconnect**: approximately 15K buses and 23K lines (Powerworld 03sfeq)
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Control subject to two constraints:

- Control an only operate in rounds 1 - 10
- Length of cascade limited to $R = 20$ rounds
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All cascades evaluated on three criteria:

- Yield
- Length of cascade ($=$ no. of rounds until stable)
- Number of outaged lines
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<th>yield %</th>
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Start

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 1

no control

yield = 100%, 24 outaged lines

control

yield 100%, 3 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 4

no control

yield = 98.06%, 339 outaged lines

control

stable, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 5

**no control**

yield = 96.39%, 592 outaged lines

**control**

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 6

no control

yield = 93.82%, 1059 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 7

no control

yield = 91.08%, 1492 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 8

**yield = 89.17%, 1975 outaged lines**

**no control**

**control**

stable at round 4, yield = 75%, 11 outaged lines

**colors**
green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 9

no control

yield = 87.17%, 2364 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload

control

stable at round 4, yield = 75%, 11 outaged lines
Round 10

**no control**

yield $= 85.16\%$, 2783 outaged lines

**control**

stable at round 4, yield $= 75\%$, 11 outaged lines

- green = normal operation
- blue = disabled by contingency
- black = outage
- red, yellow = overload
Round 11

no control

yield = 80.95%, 3318 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 12

no control

yield = 78.22%, 3686 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 13

**no control**

yield = 74.77%, 4063 outaged lines

**control**

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation  
blue = disabled by contingency  
black = outage  
red, yellow = overload
Round 14

no control

yield = 72.02%, 4356 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 15

no control

yield = 70.45%, 4619 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 16

no control

yield = 66.78%, 4976 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 17

no control

yield = 65.09%, 5471 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 18

no control

yield = 62.89%, 5806 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 19

no control

yield = 61.46%, 5907 outaged lines

control

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Round 25

stable, yield 60.78%, 5959 outaged lines

stable at round 4, yield = 75%, 11 outaged lines

green = normal operation
blue = disabled by contingency
black = outage
red, yellow = overload
Why: overloads in no-control case

![Line Overload Chart]

- **max line overload**
- **no control case**

---

[Note: The image contains a chart showing the max line overload over rounds for the no-control case.]
<table>
<thead>
<tr>
<th>r</th>
<th>No control lines out</th>
<th>yield %</th>
<th>Control 1 lines out</th>
<th>yield %</th>
<th>Control 2 lines out</th>
<th>yield %</th>
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<td>1</td>
<td>24</td>
<td>100</td>
<td>3 (.81)</td>
<td>81</td>
<td>3 (.81)</td>
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<td>0 (.92)</td>
<td>75</td>
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Why: overloads under control 1
A very different cascade
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<td>κ</td>
<td>O</td>
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<td>Y</td>
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<td>542</td>
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<td>34</td>
<td>0.99</td>
<td>0</td>
<td>995</td>
<td>78</td>
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</table>

κ = max line overload, O = outages, I = islands, Y = yield (%)
round 2

no control

control
round 3

no control

control
round 4

no control

control
round 5

no control

control
round 6

no control

control
round 7

no control

control
round 8

no control

control
round 9

no control

control
round 10

no control

control
round 11

no control  

control
round 13

no control

control
round 14

no control  control
round 15

no control

control
round 16

no control

control
round 17

no control

control
round 18

no control

control
round 19

no control

stable, yield = 75%, 2598 outaged lines

control
round 20

no control

control
round 34

no control

stable, yield = 78%, 4425 outaged lines

control

(stable at round 19, yield = 75%, 2598 outaged lines)
Stochastic models: why?

- Noise should accumulate as the cascade unfolds
- Need a better way to account for line outages near the limit
  - Deterministic rule is too unforgiving and may not match anything “real”
  - Deterministic rule is numerically unstable
**Stochastic line outage rule:**

For each round $r$, use a threshold $0 < \epsilon_r < 1$.

Given a line $k$ (notation: $\tilde{f}_k =$ moving average of $|\text{flow}|$ on line $k$),
Stochastic line outage rule:

For each round \( r \), use a threshold \( 0 < \epsilon_r < 1 \).

Given a line \( k \) (notation: \( \tilde{f}_k = \) moving average of \( |\text{flow}| \) on line \( k \)),

- \( k \) is not outaged if \( |\tilde{f}_k| < (1 - \epsilon_r)u_k \),
- \( k \) is outaged if \( |\tilde{f}_k| > u_k \), and
- \( k \) is outaged with probability \( 1/2 \), if \( (1 - \epsilon_r)u_k \leq \tilde{f}_k \leq u_k \).
Stochastic line outage rule:

For each round \( r \), use a threshold \( 0 < \epsilon_r < 1 \).

Given a line \( k \) (notation: \( \tilde{f}_k = \) moving average of \(|\text{flow}|\) on line \( k \)),

- \( k \) is not outaged if \( |\tilde{f}_k| < (1 - \epsilon_r)u_k \),
- \( k \) is outaged if \( |\tilde{f}_k| > u_k \), and
- \( k \) is outaged with probability \( 1/2 \), if \( (1 - \epsilon_r)u_k \leq \tilde{f}_k \leq u_k \).

→ Example: \( \epsilon_r = 0.01 + 0.05 \times \lfloor r/10 \rfloor \)
Technical note

► Using the above model, yield is not a differentiable function of control parameters

► As a result, no theoretical guarantee that stochastic gradients method will converge
Technical note

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▶ As a result, no theoretical guarantee that stochastic gradients method will converge

A smooth model:

Line $k$ is outaged with probability $\mathcal{F}(\tilde{f}_k/u_k)$, where

▶ $\mathcal{F}(x) \to 1$ as $x \to +\infty$,

▶ $\mathcal{F}(x) \to 0$ as $x \to 0$, 
Experiments using $\epsilon_r = 0.01 + 0.05 \times \lfloor r/10 \rfloor$

- (Second cascade discussed above)

- Uncontrolled cascade is stable at round 34, with yield = 78% and 4425 outaged lines
Experiments using $\epsilon_r = 0.01 + 0.05 \times \lfloor r/10 \rfloor$

- (Second cascade discussed above)

- Uncontrolled cascade is stable at round 34, with yield = 78% and 4425 outaged lines

- Compare to four control algorithms $c_{10}, c_{15}, c_{20}, c_{25}$

- Here, each control $c_T$ must achieve stability by round $T$, but can only shed load in rounds 1 - 10.

Why?
Experiments using $\epsilon_r = 0.01 + 0.05 \times \lfloor r/10 \rfloor$

- (Second cascade discussed above)

- Uncontrolled cascade is stable at round 34, with yield = 78\% and 4425 outaged lines

- Compare to four control algorithms $c_{10}$, $c_{15}$, $c_{20}$, $c_{25}$

- Here, each control $c_T$ must achieve stability by round $T$, but can only shed load in rounds 1 - 10.

Why?

Greater robustness is achieved by limiting the time frame
1000 runs

<table>
<thead>
<tr>
<th>Option</th>
<th>DetY</th>
<th>MaxY</th>
<th>MinY</th>
<th>AveY</th>
<th>StddY</th>
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<tr>
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<td>37.49</td>
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<td>0.00</td>
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<td>c25</td>
<td>77.23</td>
<td>54.62</td>
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<td>18.86</td>
<td>0.00</td>
<td>5.11</td>
<td>5.28</td>
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<td>(34 rounds)</td>
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