Mixed Integer Programming

A mixed-integer program (MIP) is an optimization problem of the form:

Minimize \[ c^T x \]
Subject to \[ Ax = b \]
\[ l \leq x \leq u \]
\[ \text{some } x_j \text{ integer} \]
Unit–Commitment Model

Electrical Power Industry, ERPI GS–6401, June 1989: Mixed–integer programming (MIP) is a powerful modeling tool, “[MIP models] are, however, theoretically complicated and computationally cumbersome”

**In other words**: MIP looks nice, but it can’t solve real problems
Unit–Commitment Model

• California Unit–Commitment Model

• 1999 Results (machine unknown):
  • 2–Day model: 8 hours, no progress
  • 7–Day model: 1 hour just to solve initial LP relaxation

• 2011 Results: Gurobi 4.5 ($700 desktop PC):
  • 7–Day model: proven optimal solution in 85 seconds

• Looks nice, and now can solve real problems
Model unitcal_7

Optimize a model with 48939 rows, 25755 columns and 127595 nonzeros

... 0 0 1.9450e+07 0 1362 - 1.9450e+07 - - 1s
0 0 1.9536e+07 0 1027 - 1.9536e+07 - - 3s

... H 0 0 3.118141e+07 1.9596e+07 37.2% - 17s
H 0 0 2.194163e+07 1.9596e+07 10.7% - 18s
H 0 2 1.998390e+07 1.9596e+07 1.94% - 19s
0 2 1.9596e+07 0 943 1.9984e+07 1.9596e+07 1.94% - 19s
3 5 1.9604e+07 2 1198 1.9984e+07 1.9596e+07 1.94% 771 20s
H 28 28 1.991689e+07 1.9599e+07 1.60% 646 24s
39 35 1.9612e+07 13 271 1.9917e+07 1.9599e+07 1.60% 579 25s
H 60 53 1.987731e+07 1.9599e+07 1.50% 447 26s
H 61 55 1.986798e+07 1.9599e+07 1.35% 440 26s
H 91 81 1.974075e+07 1.9599e+07 0.72% 332 27s
H 117 38 1.965010e+07 1.9605e+07 0.23% 266 29s
H 118 38 1.964944e+07 1.9605e+07 0.22% 264 29s
H 119 38 1.964934e+07 1.9605e+07 0.22% 262 29s
H 120 38 1.964931e+07 1.9605e+07 0.22% 260 29s
* 121 38 43 1.964931e+07 1.9605e+07 0.22% 258 29s
295 60 1.9621e+07 5 856 1.9649e+07 1.9607e+07 0.22% 160 31s
* 359 63 39 1.964468e+07 1.9607e+07 0.19% 149 31s
* 435 107 37 1.964403e+07 1.9607e+07 0.19% 136 32s
534 186 1.9642e+07 10 479 1.9644e+07 1.9611e+07 0.17% 125 35s
H 547 129 1.963642e+07 1.9611e+07 0.13% 124 35s
H 607 148 1.963579e+07 1.9611e+07 0.13% 116 35s
H 764 235 1.963557e+07 1.9611e+07 0.13% 100 37s
...
* 1327 412 45 1.963556e+07 1.9628e+07 0.04% 126 79s
1432 429 1.963556e+07 25 453 1.9636e+07 1.9628e+07 0.04% 121 80s

Explored 2187 nodes (236902 simplex iterations) in 84.80 seconds
Optimal solution found (tolerance 1.00e-04)
MIP Keeps Improving
Gurobi Optimization

- Founded July, 2008
- Founders: Zonghao Gu, Ed Rothberg, Bob Bixby
- Product Releases:
  - Version 1.0: May 2009
    - Performance roughly equal to CPLEX 11.0
  - Version 2.0: October 2009
  - Version 3.0: April 2010
  - Version 4.0: November 2010
  - Version 4.5: April 2011
MIP Performance – Gurobi Internal Test Set

- Version to version improvements:
  (Geometric mean runtime over ~800 models in our internal model set that take more than 100s to solve)
  - Gurobi 1.0 → 2.0: 2.2X
  - Gurobi 2.0 → 3.0: 2.9X
  - Gurobi 3.0 → 4.0: 1.3X
  - Gurobi 4.0 → 4.5: 1.8X

- Continued improvement in our ability to solve difficult MIP models
MIP Performance – Public Benchmarks

- Gurobi 4.5 vs. CPLEX 12.2.0.2 and XPRESS 7.2 (>1.0 means Gurobi is faster)

<table>
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<th>vs CPLEX 12.2.0.2</th>
<th>vs XPRESS 7.2</th>
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<tr>
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<td>P=1</td>
<td>P=4</td>
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<td>1.40X</td>
<td>–</td>
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<tr>
<td>Feasibility</td>
<td>3.57X</td>
<td>–</td>
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<tr>
<td>Infeasibility</td>
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<td>–</td>
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<tr>
<td>Pathological*</td>
<td>–</td>
<td>–</td>
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<td>MIQP</td>
<td>–</td>
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Implications of Improvements in MIP Technology For Power Industry Models
What’s Next?

• Consider an analogy – airlines
  • One of the earliest users of optimization
  • Consider…
    • Where they started
    • Where they are now
    • What we learned along the way
Where They Started

- Three high-ROI opportunities
  - Crew scheduling
  - Fleet assignment
  - Yield management (ticket prices)
- Much initial excitement about MIP (1970’s)
- Followed soon after by great disappointment
  - Quite natural modeling paradigm
  - Little success in solving practical models
  - Sound familiar?
Simplification and Custom Strategies
Simplification / Custom Strategies

• Simplication
  • Leave out important details

• Custom strategies:
  • Lagrangian relaxation, LP–based, combinatorial, custom branching, specialized cutting planes, etc.
  • Tailor–made to crew/fleet/pricing problems
    • Built from a detailed understanding of the structure of the problem
Custom Strategies

- **Pros (versus MIP model):**
  - Can be quite effective
  - Attack problems that aren’t tractable as MIPs
- **Cons (versus MIP model):**
  - Labor intensive
  - Typically quite brittle
  - Slight deviation from assumed problem structure often causes approach to fall apart
  - Often perceived as less neutral than a MIP model
Custom Approaches – Current Status

• As MIP technology improved…
  • Models routinely solved with no customization
  • Robust solutions in the presence of ‘side–constraints’
• Custom approaches no longer useful?
  • Quite the opposite:
    • Several general MIP procedures inspired by custom approaches
    • MIP solvers have assimilated the technology
Lessons Learned

- Mutual benefit from working together
  - Industry
    - Produced insights into how to solve specific models
    - Provided challenge sets
      - Representative, difficult datasets
  - MIP
    - Generalized strategies to work across a broad spectrum of MIP models
- Result: robust solutions to formerly impossible models
Datasets?

• New MIP benchmark set (MIPLIB2010)
  • Benchmark set – has a big influence of MIP algorithm development
  • Gurobi pointed out that early proposed set contained no power industry models
    • An important problem class that should be represented
  • Call went out for publicly available models
  • Result: unitcal_7 (1999)
Growing the Models
Model Growth

- Once the original problems were tractable, new MIP capabilities used to expand the model
  - Larger: more detailed representation of the problem
  - Broader: integrate multiple parts of the organization into the model
  - Faster: move from planning to real-time optimization
  - More general objectives: model non-linear objectives and constraints using piecewise-linear functions
Future Challenges
Current and Future Challenges

- Better cooperation between optimization models
  - Integration has its limits
- Dealing with uncertainty (weather, mechanical problems, etc.)
  - Techniques exist (stochastic programming, robust optimization)
  - Limited practical success so far
Moving Forward
Questions

- Insights from custom heuristics?
  - Any that might be integrated into general MIP?
- Challenge sets?
  - Can you share datasets that capture your next challenge?
- Model growth?
  - How are power industry models likely to change in the future?