DP Formulation of Least Cost Planning and an Example

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Based on Lecture Notes 18-875, Spring 2010, CMU.

Expansion planning

Problem: for a given power plant candidate set, choose when the candidate-power plant(s) should be built for all planning periods *t* such that the discounted expected capital, operating, maintenance, and penalty costs incurred during the planning horizon are minimal

$$\min_{P_g^t, s_g^t} \sum_{t=1}^{T} \rho^{t-1} \cdot \left(\sum_{g \in \text{existing G}} c_g(P_g^t) + \sum_{g \in \text{new G}} s_g^t C_g \left(P_g^{\max}\right) + C^t_{penalty} \right)$$

$$\sum_{g \in \text{old G}} P_g^t + \sum_{g \in \text{new G}} \left(s_g^t \cdot P_g^t\right) = D^t,$$

$$P_g^t \le P_g^{\max}, \quad g \in \text{old G}$$

$$P_g^t \le \left(\sum_t s_g^t \right) \cdot P_g^{\max}, \quad g \in \text{new G}$$

$$\sum_t s_g^t \le 1, \quad g \in \text{new G}$$

P_g^t is output of power plant *g* at time interval *t*,

s_g^t is a binary variable that have value 1 if new power plant *g* is built at time interval *t* and 0 otherwise,

K_g is maximum output of new power plant *g*,

$P_g^{\max}$ is maximum output of old power plant *g*,

c_g is power plant *g* operational costs,

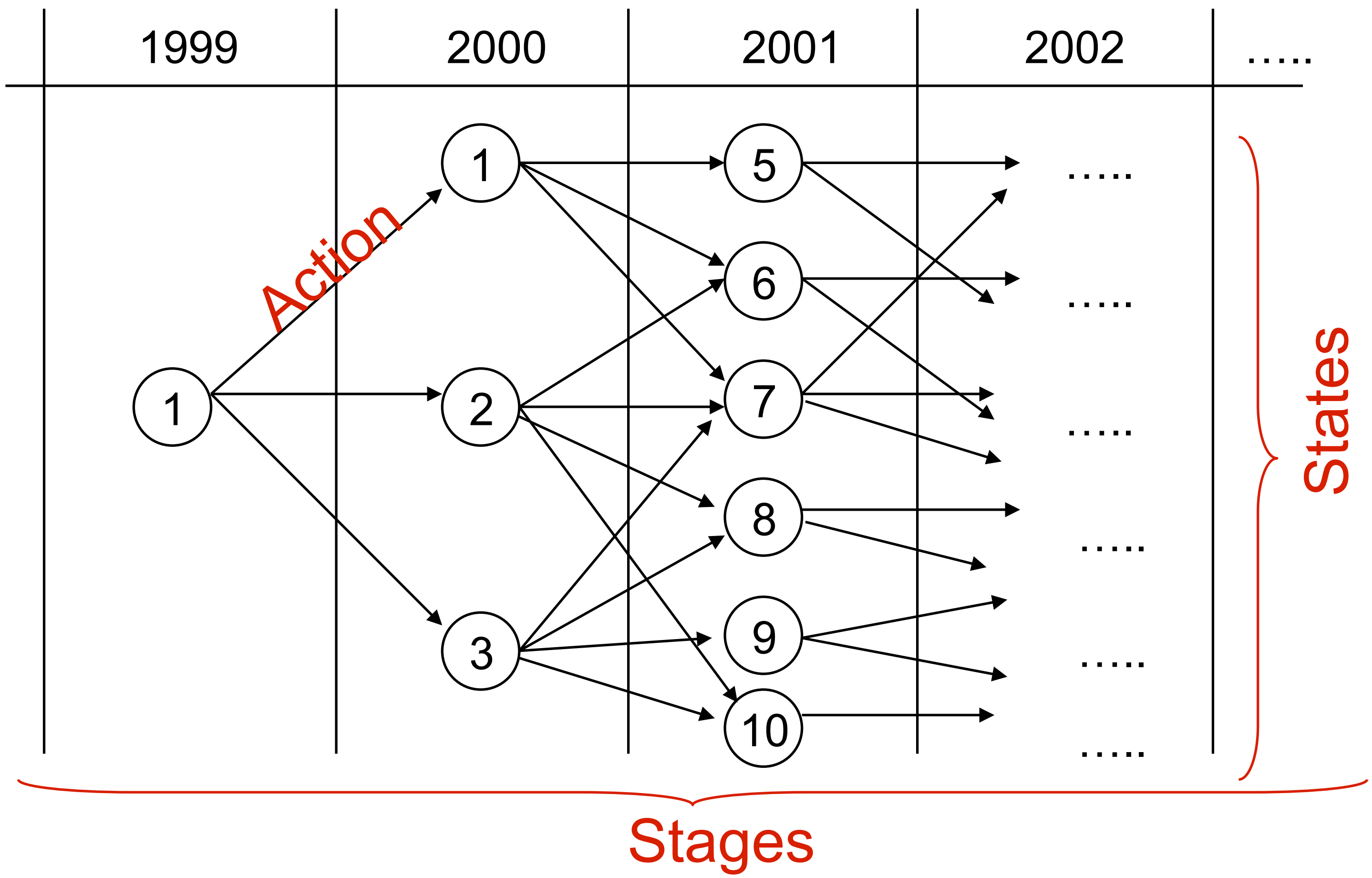
C_g is power plant *g* capital cost and fixed operating costs,

D_t is demand at time interval *t*,

ρ is discount factor,

T is planning horizon.

Decision Tree



Optimal path

$$C^* = \min_{p \in \omega} \left\{ C_x = \sum_{u \in \eta_p} \left\{ \left(C_I\right)_{s_1 \to s_2}^{(t-1) \to t} + \left(c_{o\&m}\right)_{s_2}^{t} + \left(C_{penalty}\right)_{s_2}^{t} \right\} \right\}$$

$\left(C_I\right)_{s_1 \to s_2}^{(t-1) \to t}$ Investment cost for moving from state s_1 in year *(t-1)* to state s_2 in year *t* (Investment cost * Rate of Return)

$\left(c_{o\&m}\right)_{s_2}^{t}$ O&M cost for at state s_2 in year *t* (Variable O&M + Marginal Fuel Cost)

$\left(C_{penalty}\right)_{s_2}^{t}$ Penalty cost for at state s_2 in year *t*

$p;\ \omega$ Path; Set of all possible paths

$u;\ \eta_p$ Branch; Set of all branches that belongs to path *p*

$C^*;\ C_p$ Optimal strategy cost; Total cost for path *p*

DP – Tabular Approach

- **States**: State is existence of the power plants, both existing and new candidate units. This variable is chosen as a state because the existence of new power plant can be controlled by the action of building or not building a unit.

$$\mathbf{s}_i = \begin{bmatrix} s_{i1} & s_{i2} & \cdots & s_{in} \end{bmatrix} \qquad s_{ij} = \begin{cases} 1 & \text{if unit "j" exists in the state "i"} \\ 0 & \text{if unit "j" does not exists in the state "i"} \end{cases}$$

- **Actions**: Build the candidate-power plant(s) or do nothing.

$$\mathbf{x}_i = \begin{bmatrix} x_{i1} & x_{i2} & \cdots & x_{ik} \end{bmatrix} \qquad x_{ij} = \begin{cases} 1 & \text{if unit "j" will be built with the action "i"} \\ 0 & \text{if unit "j" will not be built with the action "i"} \end{cases}$$

- **States and actions tables**:

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_m \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_m \end{bmatrix}$$

Reward function

If the system is in state $\mathbf{s}_i^t$ and action $\mathbf{x}_j^t$ is taken then in period *t* the reward will be:

$$f(\mathbf{s}_i^t, \mathbf{x}_j^t, t) = \begin{cases} \mathbf{OC}(\mathbf{s}_i^t, t) + \mathbf{C}(\mathbf{x}_j^t, t) & \mathbf{s}_i^{t+1} - \mathbf{s}_i^t > 0 & \text{(new power plant is built)} \\ \mathbf{OC}(\mathbf{s}_i^t, t) & \mathbf{s}_i^{t+1} = \mathbf{s}_i^t & \text{(power plant is not built)} \\ \infty & \mathbf{s}_i^{t+1} - \mathbf{s}_i^t < 0 & \text{(power plant does not exsists in the next state)} \end{cases}$$

Expected operation and maintenance costs:

$$\mathbf{OC}(i,t) = \mathbf{OC}(\mathbf{s} = \mathbf{s}_i^t, t) = \min \sum_{g \in \mathbf{s}_i^t} c_g\left(P_g(t)\right)$$

$$\sum_{g \in \mathbf{s}_i^t} P_g(t) = D^t,$$

$$P_g(t) \le \mathbf{s}_i^t \cdot P_g^{\max}$$

$c_g(P_g)$ = Variable O&M + Marginal Fuel Cost

Capital costs:

$$\mathbf{C}_g = RoR \cdot \mathbf{C}_I$$

$$C(\mathbf{x}_j^t, t) = \mathbf{x}_j^t \cdot \mathbf{C}_g = \mathbf{x}_j^t \cdot \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_k \end{bmatrix} = \sum_{x_{ij}^t \in \mathbf{x}_j^t} x_{ij}^t \cdot C_i$$

DP – Tabular Approach

- **Reward table**

$$\mathbf{F}(\text{state } i, \text{action } j, t) = \mathbf{F}(i, j, t) = \mathbf{F}(\mathbf{s} = \mathbf{s}_i, \mathbf{x} = \mathbf{x}_j, t) = \begin{bmatrix} f_{11}^t & f_{12}^t & \cdots & f_{1m}^t \\ f_{21}^t & f_{22}^t & \cdots & f_{2m}^t \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1}^t & f_{n2}^t & \cdots & f_{nm}^t \end{bmatrix}$$

$$f_{ij}^t = f(\mathbf{s} = \mathbf{s}_i^t, \mathbf{x} = \mathbf{x}_j^t, t) = \mathbf{OC}(\mathbf{s}_i^t, t) + \mathbf{C}(\mathbf{x}_j^t, t)$$

- **Transition table:** states correspond to rows and the actions correspond to columns of the table and an element is the next state index from the state matrix:

$$\mathbf{TR}(\text{state } i, \text{action } j) = \mathbf{TR}(i, j) = \mathbf{TR}(\mathbf{s} = \mathbf{s}_i, \mathbf{x} = \mathbf{x}_j) = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{nm} \end{bmatrix}$$

$$t_{ij} = \text{index of } \left(\mathbf{s}_k = \mathbf{s}_i \vee \mathbf{x}_j\right) \text{ in } \mathbf{S}$$

Example

Old power plants

Type:	Coal	
Capacity:	15 GW	
Capital Cost per MW:	$1,200,000/MW	
Capital Cost per Unit:	$0	(existing unit - is already paid off)
Variable O&M:	$5/MWh	
Heat Rate:	10,000 Btu/kWh	

Annual Fuel Price	Year	Marginal Fuel Cost
$1.36/MBtu	(1994)	$13.6000/MWh
$1.32/MBtu	(1995)	$13.2000/MWh
$1.29/MBtu	(1996)	$12.9000/MWh
$1.27/MBtu	(1997)	$12.7000/MWh
$1.25/Mbtu	(1998)	$12.5000/MWh

Type:	Oil	
Capacity:	6 GW	
Capital Cost per MW:	$350,000/MW	
Capital Cost per Unit:	$0	(existing unit - is already paid off)
Variable O&M:	$10/MWh	
Heat Rate:	10,470 Btu/kWh	

Annual Fuel Price	Year	Marginal Fuel Cost
$2.40/MBtu	(1994)	$25.1280/MWh
$2.60/MBtu	(1995)	$27.2220/MWh
$3.01/MBtu	(1996)	$31.5147/MWh
$2.76/MBtu	(1997)	$29.2113/MWh
$2.38/Mbtu	(1998)	$21.6729/MWh

Type:	Gas	
Capacity:	1.7 GW	
Capital Cost per MW:	$500,000/MW	
Capital Cost per Unit:	$0	(existing unit - is already paid off)
Variable O&M:	$10/MWh	
Heat Rate:	6,500 Btu/kWh	

Annual Fuel Price	Year	Marginal Fuel Cost
$2.23/MBtu	(1994)	$14.4950/MWh
$1.98/MBtu	(1995)	$12.8700/MWh
$2.64/MBtu	(1996)	$17.1600/MWh
$2.76/MBtu	(1997)	$17.9400/MWh
$2.38/Mbtu	(1998)	$15.4700/MWh

New power plants

Type:	Coal
Capacity:	3 GW
Operational Life:	40 years
Capital Cost per MW:	$1,200,000/MW
Capital Cost per Unit:	$3,600,000,000
Variable O&M:	$5/MWh
Heat Rate:	9,380 Btu/kWh

Annual Fuel Price	Year	Marginal Fuel Cost
$1.36/MBtu	(1994)	$12.7568/MWh
$1.32/MBtu	(1995)	$12.3816/MWh
$1.29/MBtu	(1996)	$12.1002/MWh
$1.27/MBtu	(1997)	$11.9126/MWh
$1.25/Mbtu	(1998)	$11.7250/MWh

Type:	Oil
Capacity:	5 GW
Operational Life:	30 years
Capital Cost per MW:	$350,000/MW
Capital Cost per Unit:	$1,750,000,000
Variable O&M:	$10/MWh
Heat Rate:	14,430 Btu/kWh

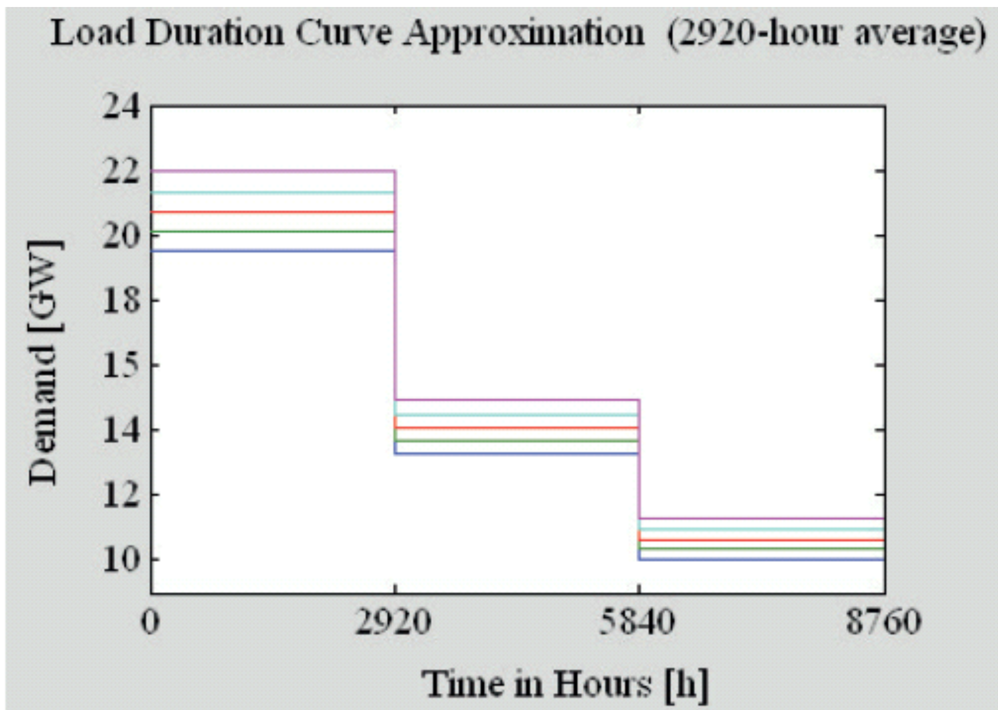
Annual Fuel Price	Year	Marginal Fuel Cost
$2.40/MBtu	(1994)	$34.6320/MWh
$2.60/MBtu	(1995)	$37.5180/MWh
$3.01/MBtu	(1996)	$43.4343/MWh
$2.76/MBtu	(1997)	$40.2597/MWh
$2.38/Mbtu	(1998)	$29.8701/MWh

Type:	Gas
Capacity:	3.8 GW
Operational Life:	30 years
Capital Cost per MW:	$500,000/MW
Capital Cost per Unit:	$1.900,000,000
Variable O&M:	$10/MWh
Heat Rate:	6,500 Btu/kWh

Annual Fuel Price	Year	Marginal Fuel Cost
$2.23/MBtu	(1994)	$14.4950/MWh
$1.98/MBtu	(1995)	$12.8700/MWh
$2.64/MBtu	(1996)	$17.1600/MWh
$2.76/MBtu	(1997)	$17.9400/MWh
$2.38/Mbtu	(1998)	$15.4700/MWh

Example -Load duration curve

- **Forecasted Demand** (GW): Basic load curve is ISO NE 1994. Load growth is assumed 3% per year. Load duration curve (LDC) is calculate as an a 2920-hour average, except for the first interval where the maximum 2920-hour maximum is used.



Example - States and Action Tables

- States table

$$\mathbf{S}(state, PowerPlant) = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{array} \begin{array}{c} \begin{array}{cccccc} \text{p1} & \text{p2} & \text{p3} & \text{ppc1} & \text{ppg1} & \text{ppo1} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

- Action table

$$\mathbf{X}(action, PowerPlant) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{array} \begin{array}{c} \begin{array}{ccc} \text{ppc1} & \text{ppg1} & \text{ppo1} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Example -Transition Table

$$\mathbf{TR} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 2 & 4 & 4 & 6 & 6 & 8 & 8 \\ 3 & 4 & 3 & 4 & 7 & 8 & 7 & 8 \\ 4 & 4 & 4 & 4 & 8 & 8 & 8 & 8 \\ 5 & 6 & 7 & 8 & 5 & 6 & 7 & 8 \\ 6 & 6 & 8 & 8 & 6 & 6 & 8 & 8 \\ 7 & 8 & 7 & 8 & 7 & 8 & 7 & 8 \\ 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \end{bmatrix} \end{array}$$

$$s_5 \vee x_1 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} & & & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} = s_5 \Rightarrow \text{index in } \mathbf{S} = 5$$

$$s_5 \vee x_2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} & & & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} = s_6 \Rightarrow \text{index in } \mathbf{S} = 6$$

$$s_5 \vee x_8 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} & & & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = s_8 \Rightarrow \text{index in } \mathbf{S} = 8$$

Example - Year 1

State I: 1 1 1 0 0 0

Energy Produced

Eo1=(10,032+13,256+15,000) 2920=111,800,645 MWh

Eo2=2,807 2920=8,196,440 MWh

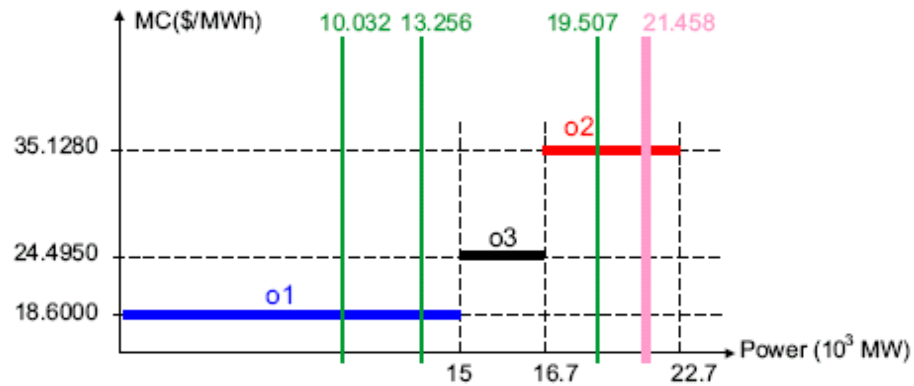
Eo3=1,700 2920=4,964,000 MWh

Cost Regulated

Co1=111,800,645 18.6 = $ 2,079,491,990

Co2=8,196,440 35.128 = $ 287,924,544

Co3=4,964,000 24.495 = $ 121,593,180



State II: 1 1 1 1 0 0 0

Energy Produced

Eo1=(7,032+10,256+15,000) 2920=94,280,645 MWh

Eo2=0 MWh

Eo3=1,507 2920=4,400,440 MWh

Ec1=3 3,000 2920=26,280,000 MWh

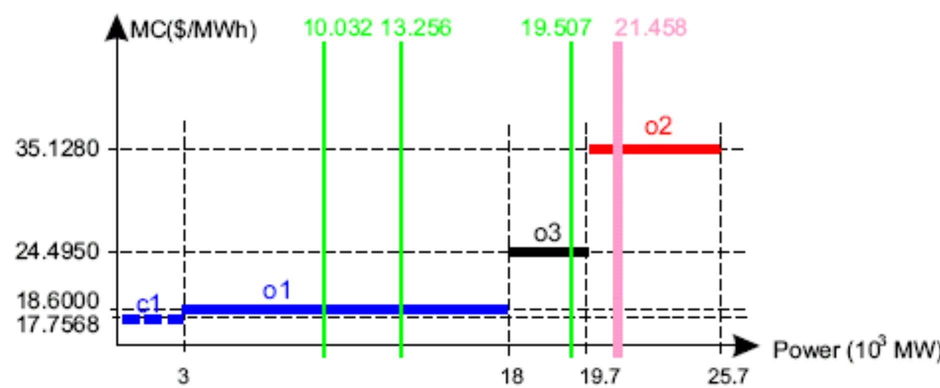
Cost Regulated

Co1=94,280,645 18.6000 = $ 1,753,619,990

Co2= $ 0

Co3=4,400,440 24.4950 = $ 107,788,778

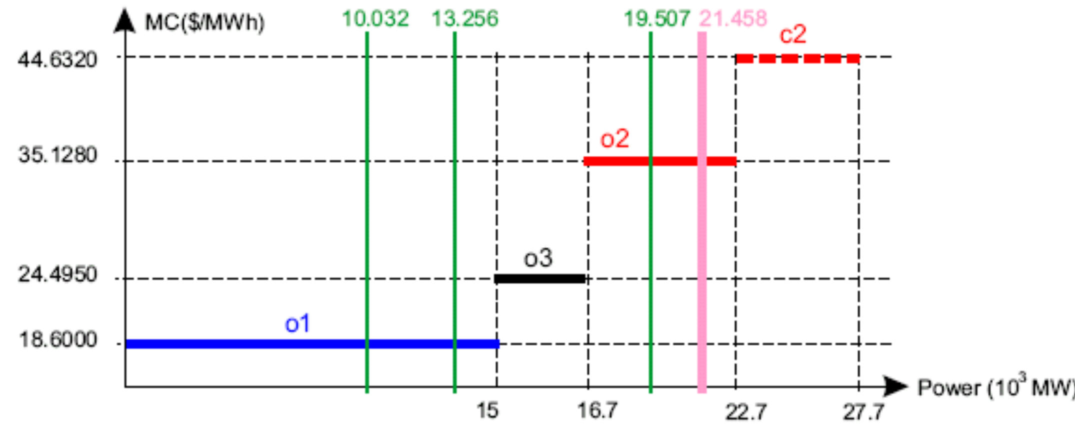
Cc1=26,280,000 17.7568 = $ 466,648,704



Example - Year 1 (cont'd)

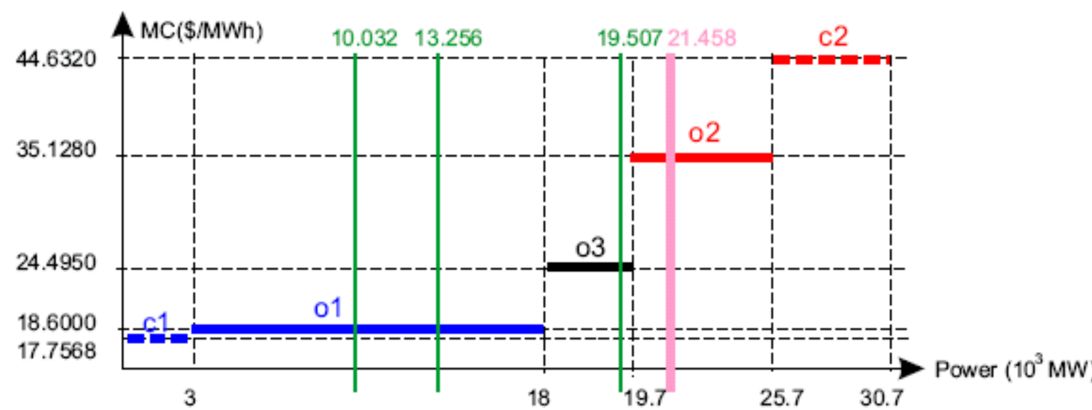
State III: 1 1 1 0 1 0

Cost Regulated
Co1=94,280,645 18.6000 = $ 1,753,619,990
Co2= $ 0
Co3= 4,400,440 24.495 = $ 107,788,778
Cc1=26,280,000 17.7568 = $ 466,648,704
Cc2= $ 0



State IV: 1 1 1 1 1 0

Cost Regulated
Co1=94,280,645 18.6000 = $ 1,753,619,990
Co2= $ 0
Co3= 4,400,440 24.495 = $ 107,788,778
Cc1=26,280,000 17.7568 = $ 466,648,704
Cc2= $ 0



Example - Year 1 (cont'd)

State V: 1 1 1 0 0 1

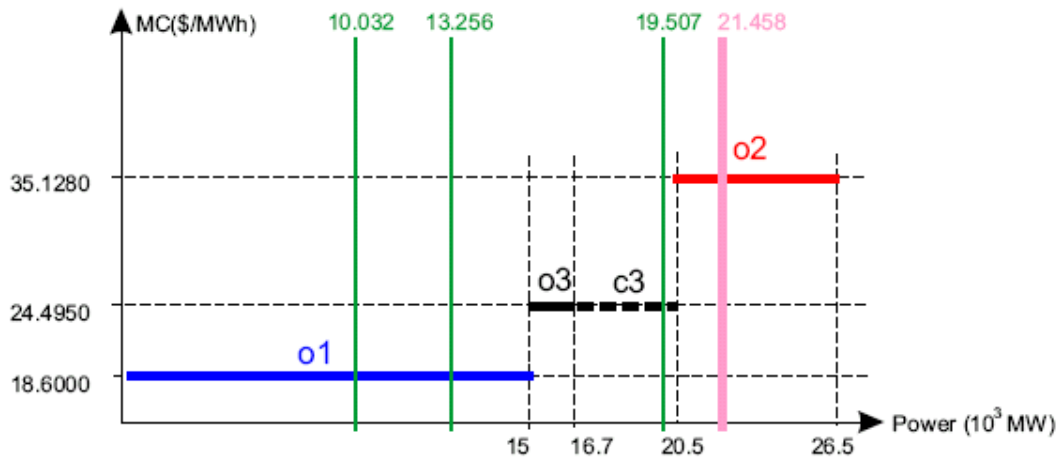
Cost Regulated

Co1=111,800,645 18.6 = $ 2,079,491,990

Co2=$ 0

Co3=4,964,000 24.495 = $ 121,593,180

Cc3=8,196,440 24.495 = $ 200,771,798



State VI: 1 1 1 1 0 1

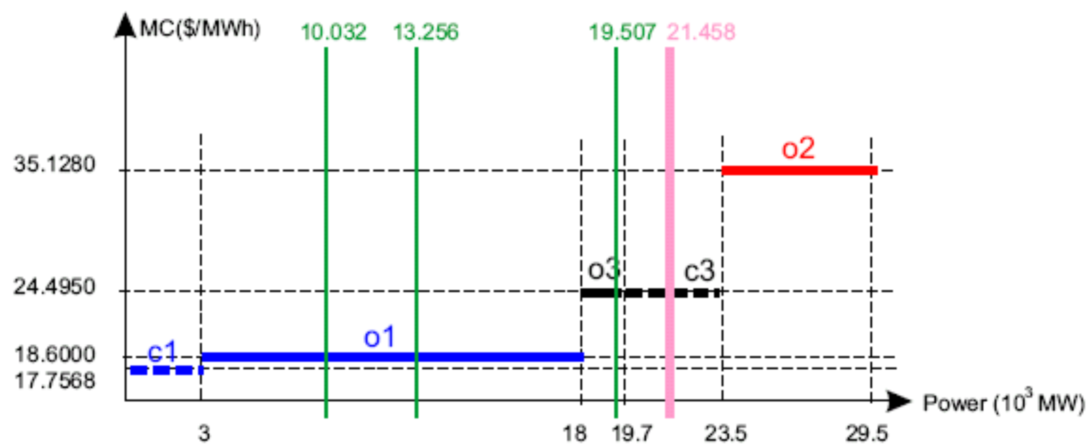
Cost Regulated

Co1=94,280,645 18.6 = $ 1,753,619,990

Co2= Cc3 = $ 0

Co3=4,400,440 24.495 = $ 107,788,778

Cc1=26,280,000 18.6 = $ 466,648,704



Example - Year 1 (cont'd)

State VII: 1 1 1 0 1 1

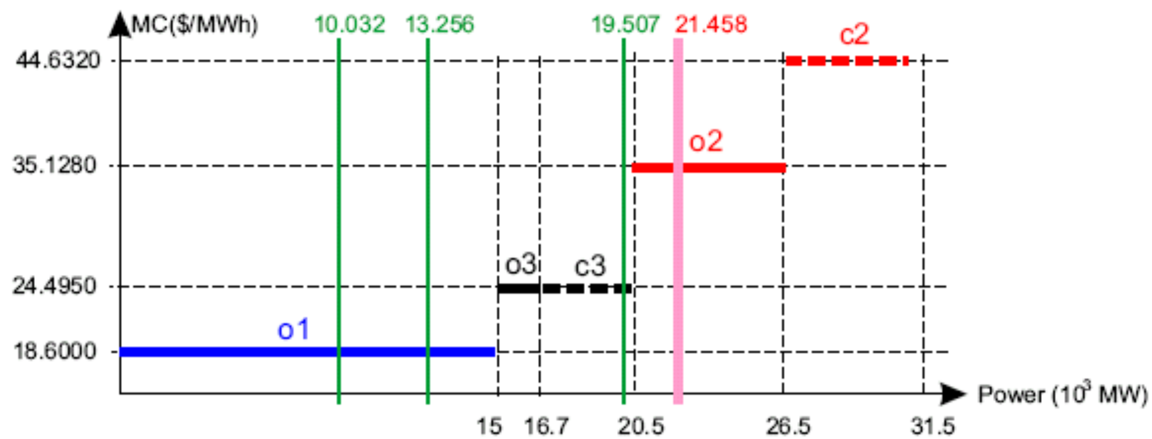
Cost Regulated

Co1=111,800,645 18.6 = $ 2,079,491,990

Co2= Cc2 = $ 0

Co3=4,964,000 24.495 = $ 121,593,180

Cc3=8,196,440 24.495 = $ 200,771,798



State VIII: 1 1 1 1 1 1

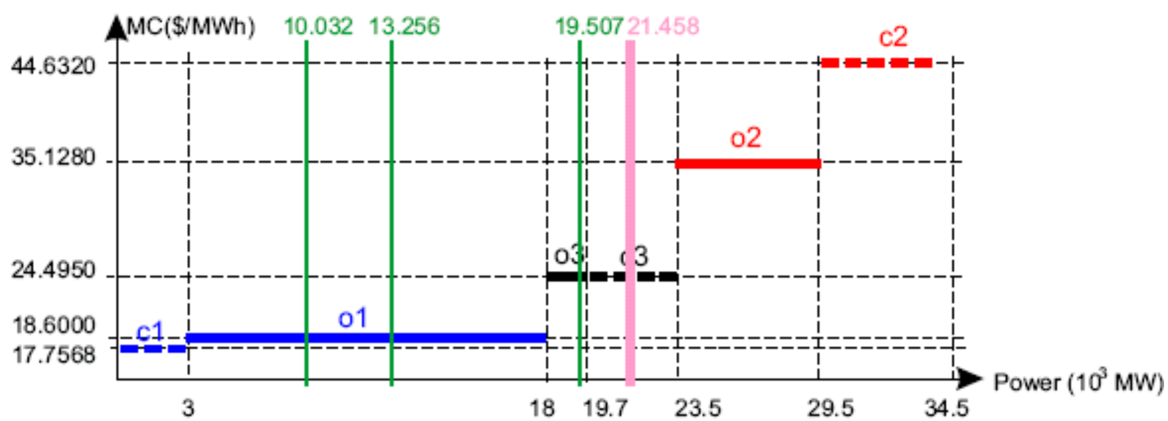
Cost Regulated

Co1=94,280,645 18.6 = $ 1,753,619,990

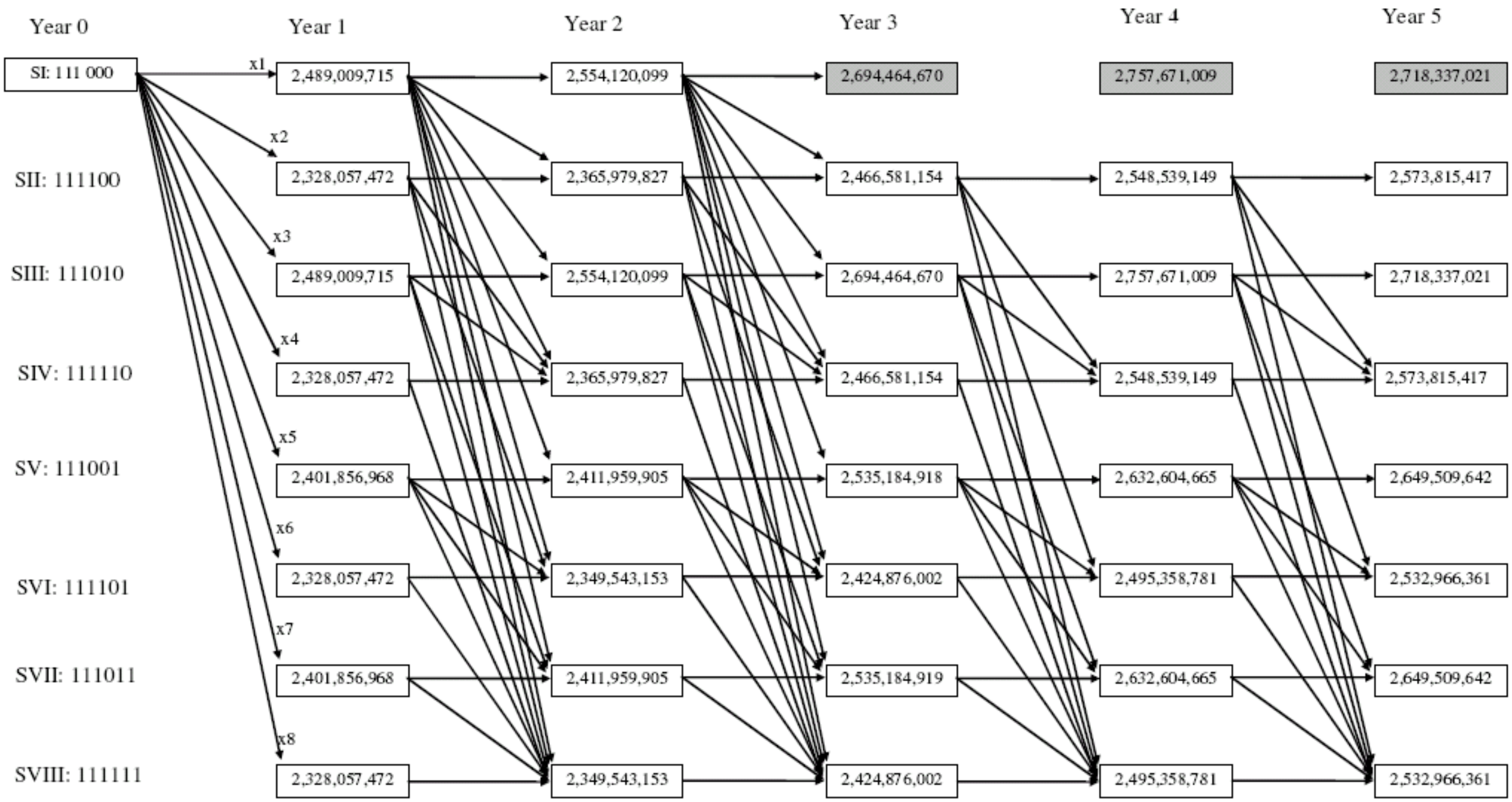
Co2= Cc2 = Cc3 = $ 0

Co3=4,400,440 24.495 = $ 107,788,778

Cc1=26,280,000 18.6 = $ 466,648,704



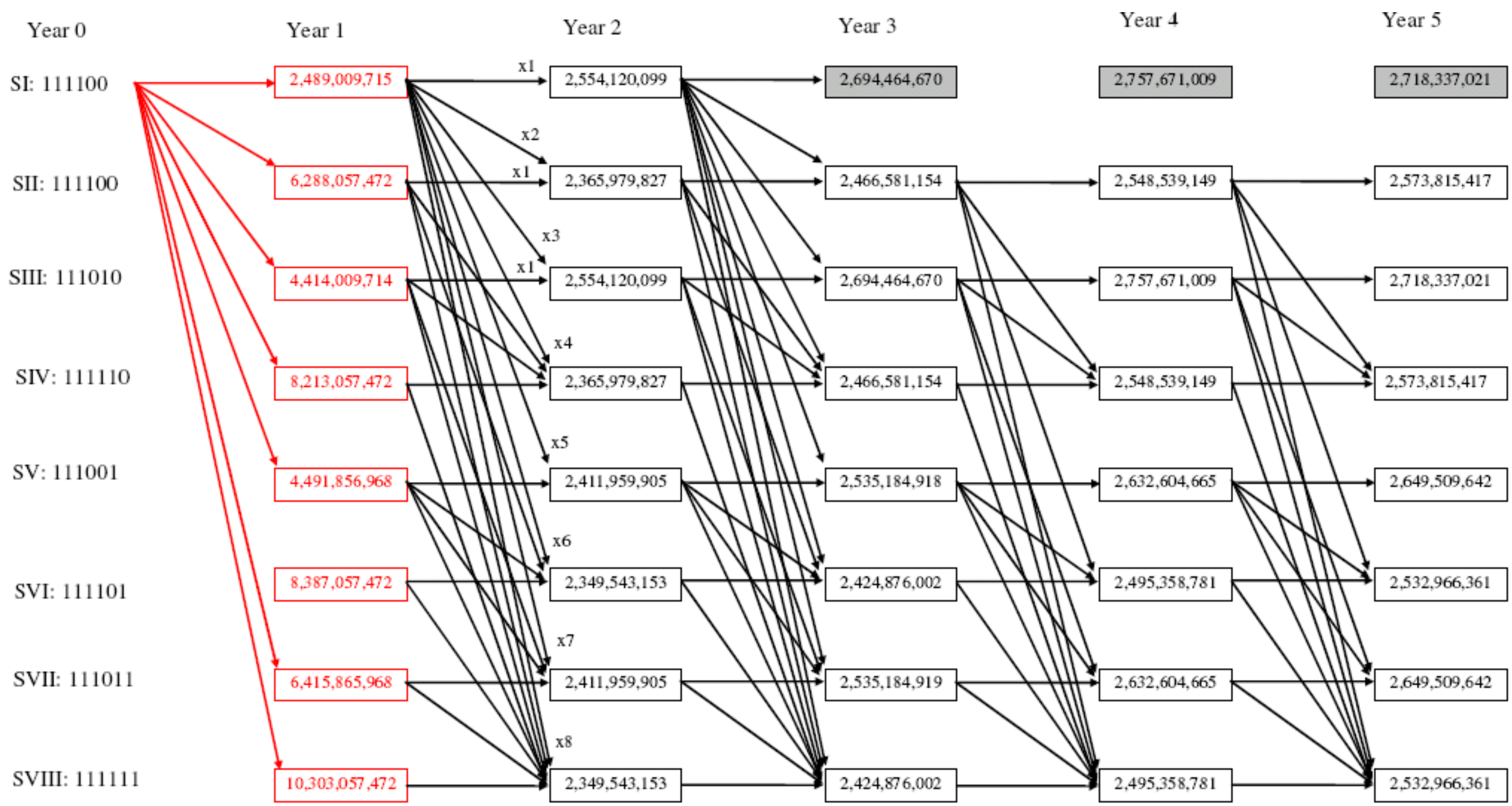
Example - Decision tree



Capital Cost for the actions: x1: \$ 0 x3: \$ 1,925,000,000 x5: \$ 2,090,000,000 x7: \$ 4,015,000,000

x2: \$ 3,960,000,000 x4: \$ 5,885,000,000 x6: \$ 6,050,000,000 x8: \$ 7,975,000,000

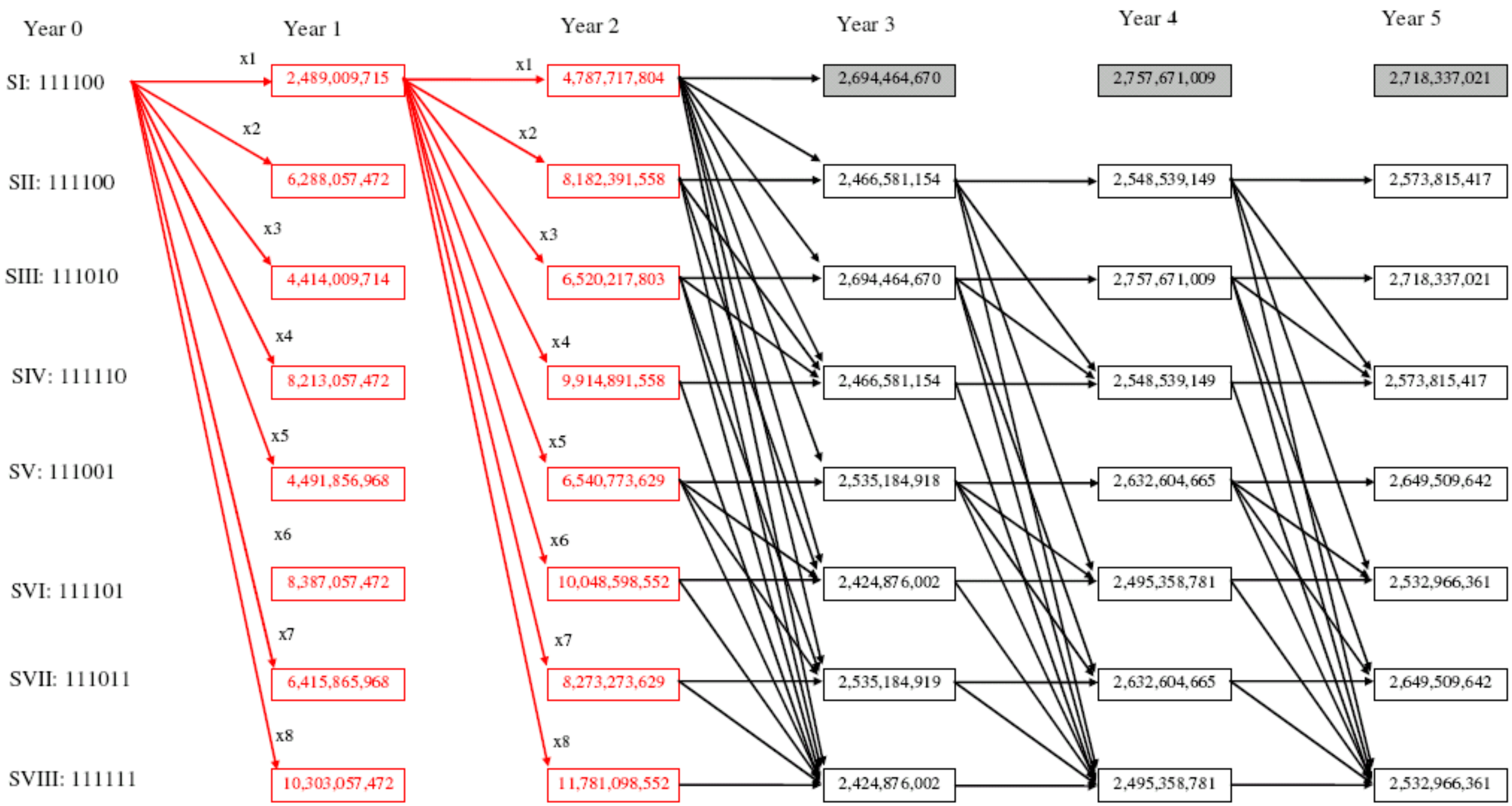
Example - Decision tree



Capital Cost for the actions:

x1: $ 0
x2: $ 3,960,000,000
x3: $ 1,925,000,000
x4: $ 5,885,000,000
x5: $ 2,090,000,000
x6: $ 6,050,000,000
x7: $ 4,015,000,000
x8: $ 7,975,000,000

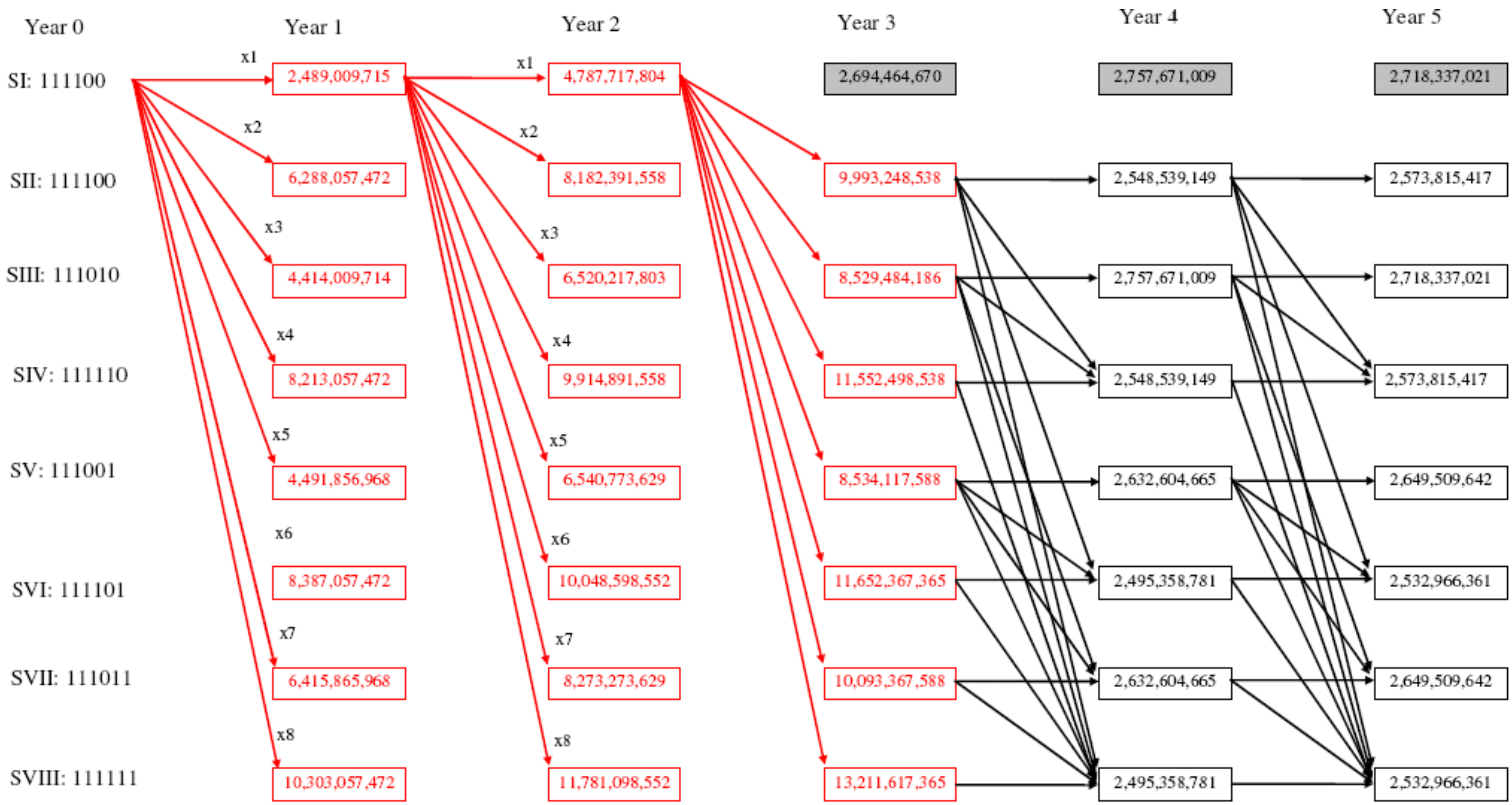
Example - Decision tree



Capital Cost for the actions: x1: $ 0 x3: $ 1,925,000,000 x5: $ 2,090,000,000 x7: $ 4,015,000,000

x2: $ 3,960,000,000 x4: $ 5,885,000,000 x6: $ 6,050,000,000 x8: $ 7,975,000,000

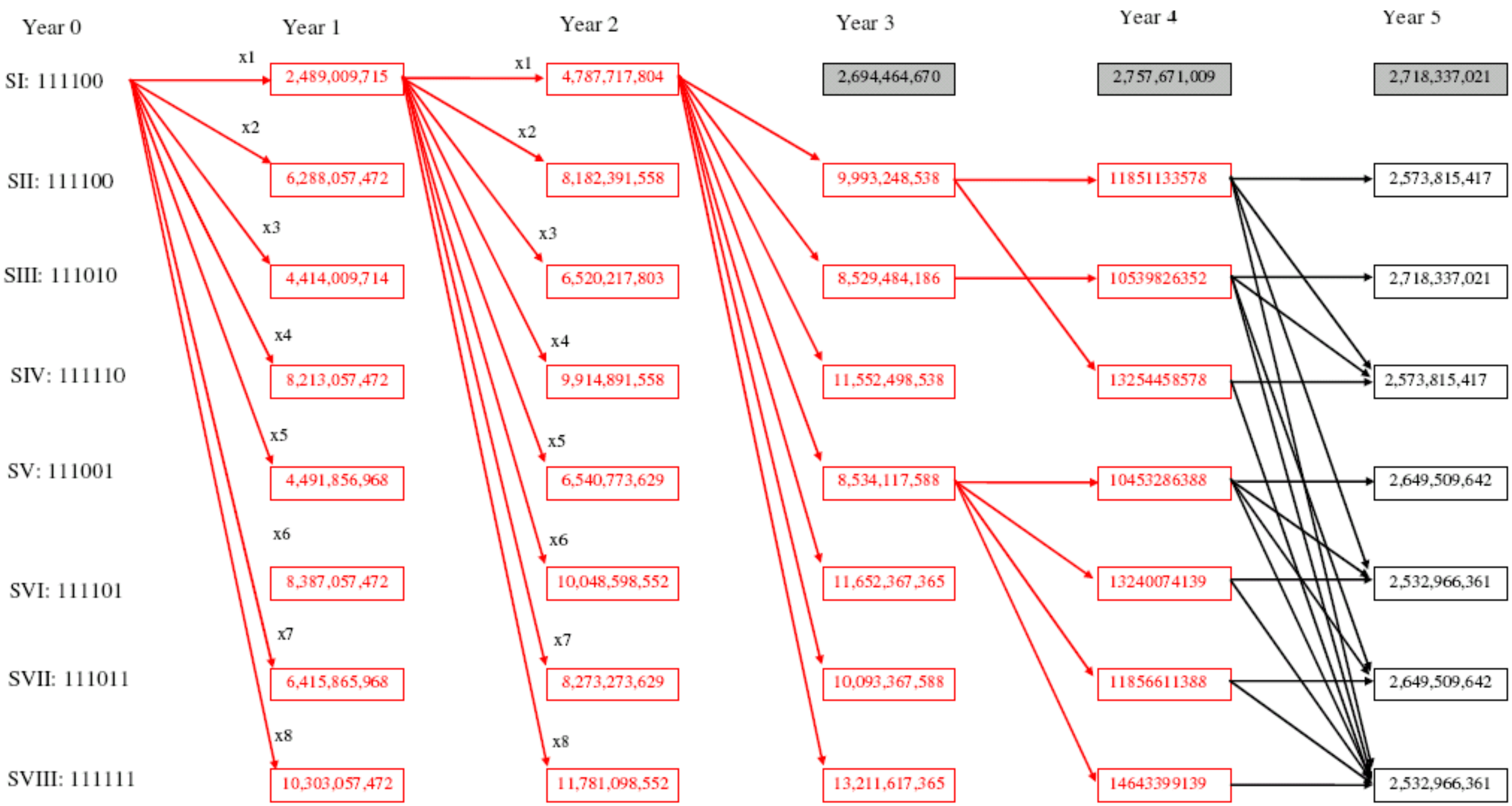
Example - Decision tree



Capital Cost for the actions: x1: $ 0 x3: $ 1,925,000,000 x5: $ 2,090,000,000 x7: $ 4,015,000,000

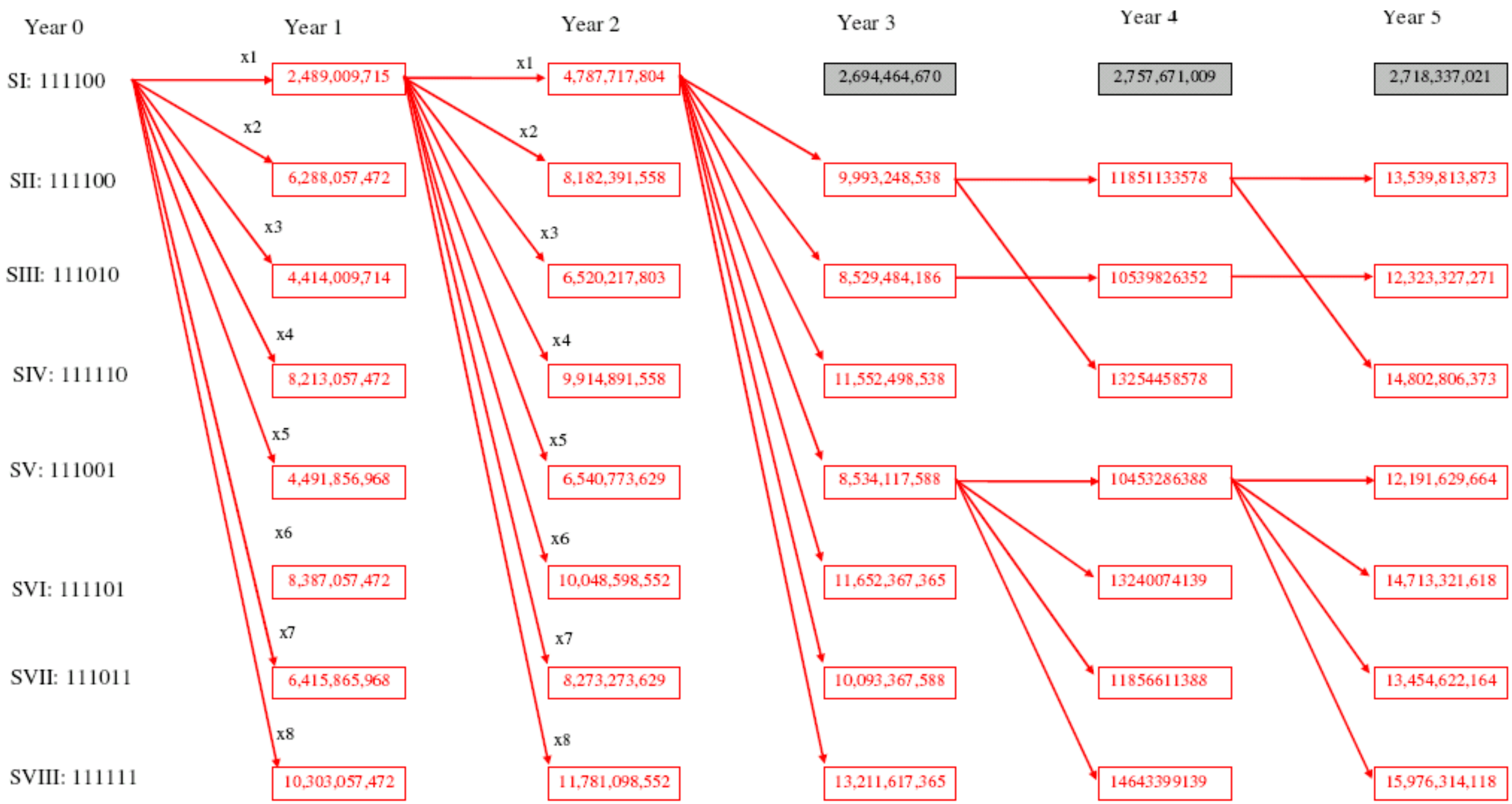
x2: $ 3,960,000,000 x4: $ 5,885,000,000 x6: $ 6,050,000,000 x8: $ 7,975,000,000

Example - Decision tree



Capital Cost for the actions: x1: $ 0; x2: $ 3,960,000,000; x3: $ 1,925,000,000; x4: $ 5,885,000,000; x5: $ 2,090,000,000; x6: $ 6,050,000,000; x7: $ 4,015,000,000; x8: $ 7,975,000,000

Example - Decision tree

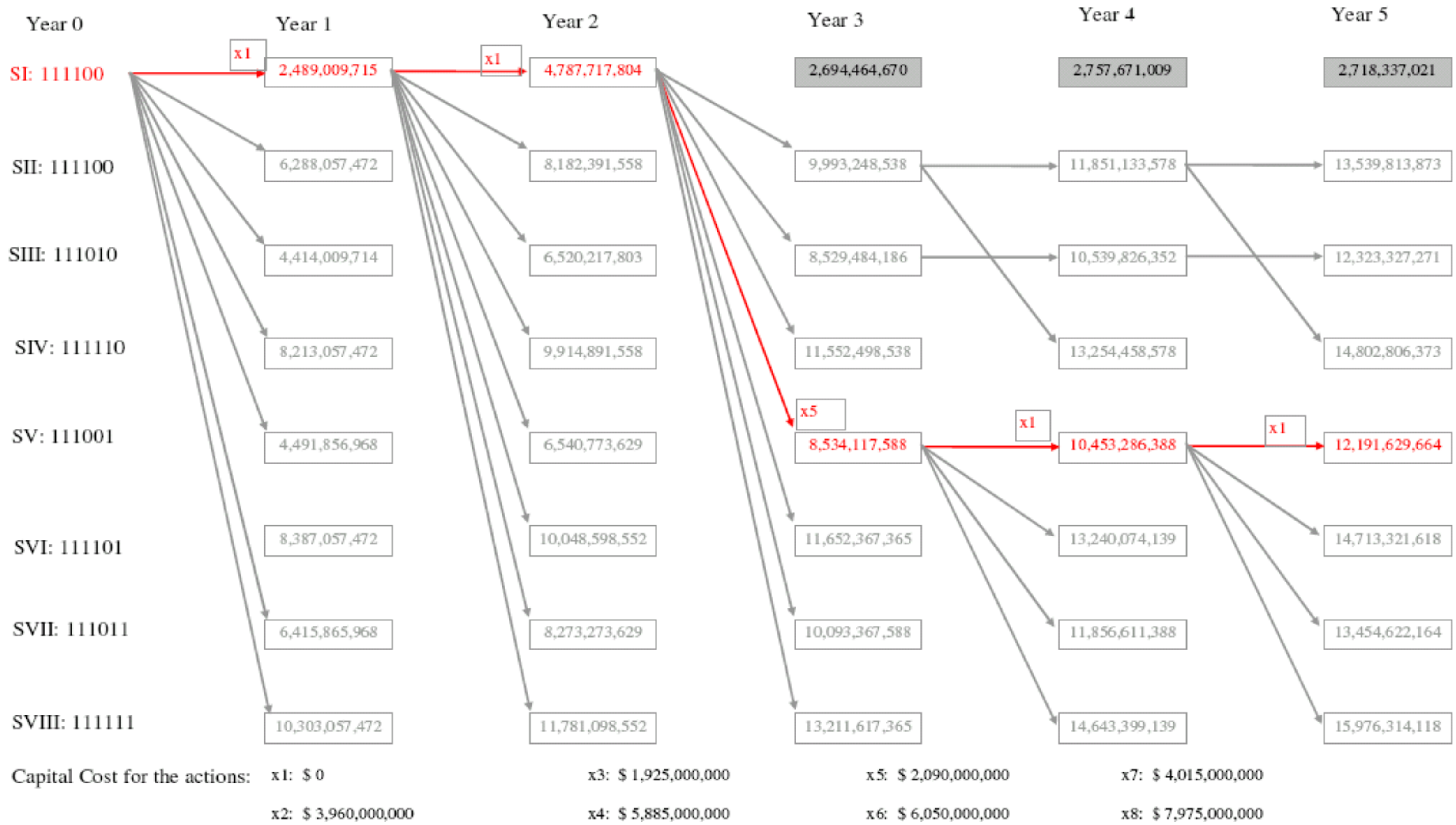


Capital Cost for the actions:

x1: $ 0
x2: $ 3,960,000,000
x3: $ 1,925,000,000
x4: $ 5,885,000,000
x5: $ 2,090,000,000
x6: $ 6,050,000,000
x7: $ 4,015,000,000
x8: $ 7,975,000,000

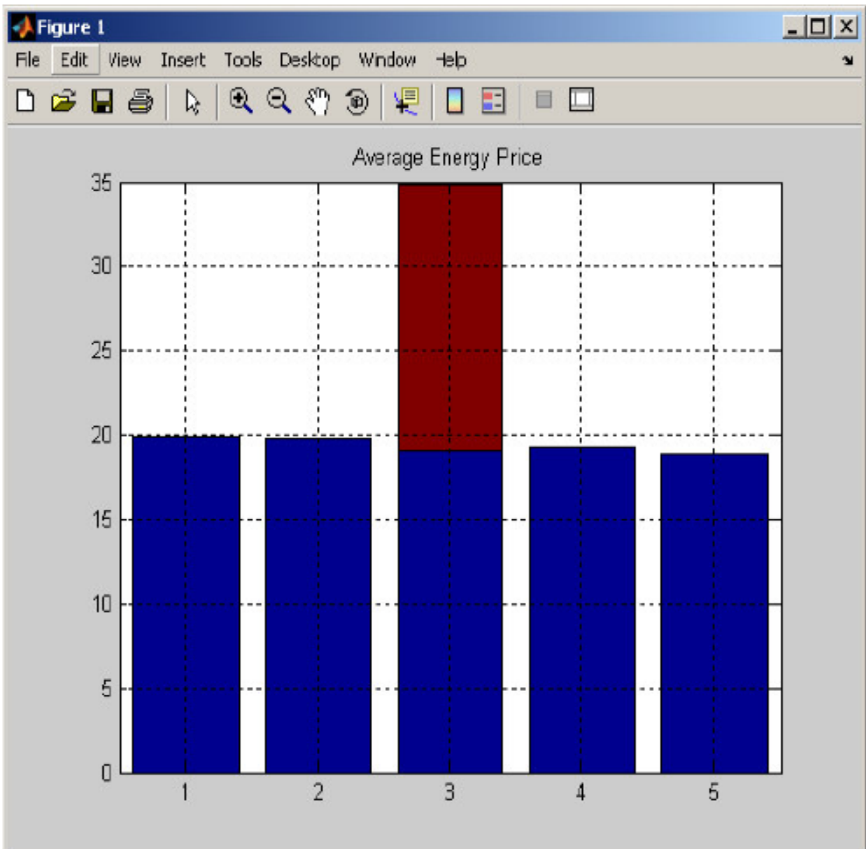
Example – Optimal strategy

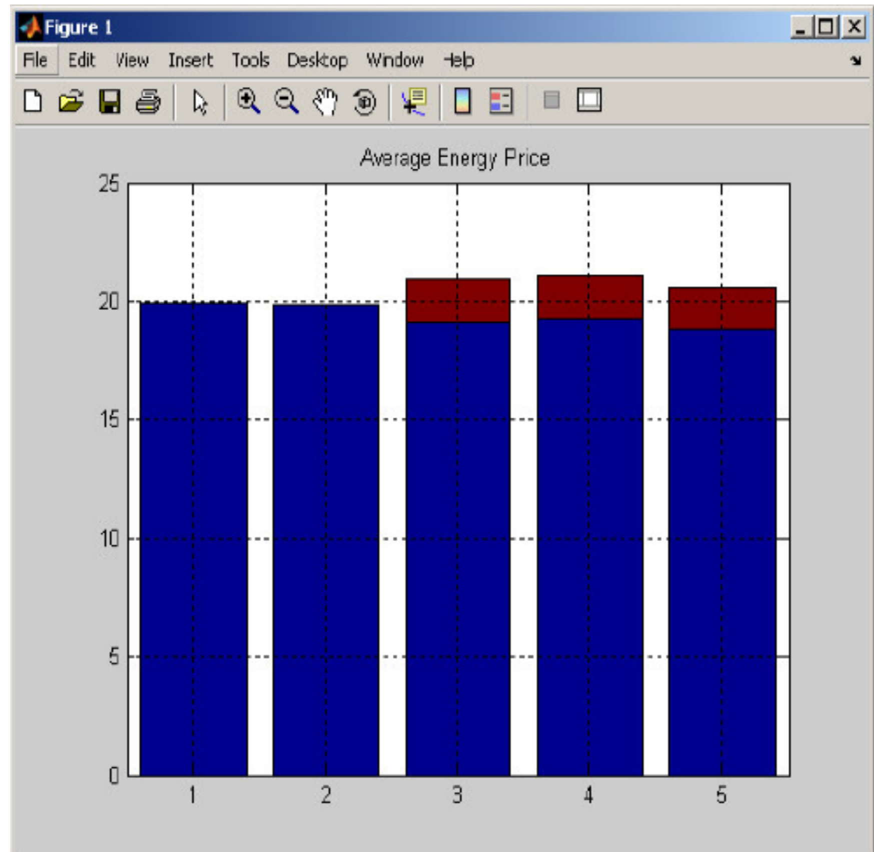
Optimal action policy is: x1; x1; x5; x1; x1 and the optimal states are S(I,1), S(I,2), S(V,3), S(V,4), S(V,5). Total plan cost is $12,191,629,664.



Example – Average Electricity Price

To earn 10% rate of return on the investment average electricity price is:





a) One time investment

b) Uniform-investment over 30years

Summary

- The example given is for regulated least -cost-planning
- See Marija Prica, PhD ECE CMU, June 2010 for the results on Centralized Peak Load Pricing, as well as Interactive Planning Framework using the same numerical approach.
- Contact: milic@ece.cmu.edu for more info on this.